MATH 267 Fall 2026 - Class Notes

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Summary: Today's class introduced basic set operations, including union, intersection, complement, and set difference. We also discussed De Morgan's laws, the double-inclusion method for proving set equality, and symmetric difference. Several examples and proofs were used to show how these operations and identities work.

Notes:

Basic Set Operations

Let X be the universal set and let

$$A, B \subseteq X.$$

For the main example, let

$$A = \{1, 2, 3, 4\}, \qquad B = \{3, 4, 5, 6\}.$$

Union

The union of A and B contains everything that is in A, in B, or in both sets.

$$A \cup B = \{x \in X : x \in A \text{ or } x \in B\}.$$

For example,

$$A = \{1, 2, 3, 4\}, \qquad B = \{3, 4, 5, 6\}.$$

Then

$$A \cup B = \{1, 2, 3, 4, 5, 6\}.$$

The elements 3 and 4 are not repeated because sets do not contain duplicate elements.

Intersection

The intersection contains everything that belongs to both A and B.

$$A \cap B = \{x \in X : x \in A \text{ and } x \in B\}.$$

For our example,

$$A \cap B = \{3, 4\}.$$

Universal Set and Complement

The universal set is the largest set under consideration. All sets being discussed are subsets of the universal set.

Suppose

$$X = \{1, 2, 3, 4, 5, 6, 7\}.$$

If

$$A = \{1, 2, 3, 4\},$$

then

$$A^c = \{5, 6, 7\}.$$

Thus, A^c contains everything in the universe that is not in A.

Similarly, since

$$B = \{3, 4, 5, 6\},$$

we have

$$B^c = \{1, 2, 7\}.$$

Set Difference

The set difference $A \setminus B$ contains the elements that belong to A but do not belong to B.

$$A \setminus B = \{x \in X : x \in A \text{ and } x \notin B\}.$$

The symbol $\setminus$ can be thought of as "set minus."

For example,

$$A \setminus B = \{1, 2\},$$

while

$$B \setminus A = \{5, 6\}.$$

Set difference can also be written as

$$A \setminus B = A \cap B^c.$$

For example,

$$B^c = \{1, 2, 7\}.$$

Therefore,

$$A \cap B^c = \{1, 2, 3, 4\} \cap \{1, 2, 7\} = \{1, 2\}.$$

Thus,

$$A \setminus B = A \cap B^c.$$

Important Set Identities

Some important identities are

$$A \cup \varnothing = A,$$

$$A \cap \varnothing = \varnothing,$$

$$A \cup A = A,$$

$$A \cap A = A,$$

and

$$(A^c)^c = A.$$

For example, if

$$A = \{1, 2, 3, 4\},$$

then

$$A \cup \varnothing = A$$

and

$$A \cap \varnothing = \varnothing.$$

De Morgan's Laws

De Morgan's laws are

$$(A \cup B)^c = A^c \cap B^c$$

and

$$(A \cap B)^c = A^c \cup B^c.$$

An easy way to remember these laws is that taking the complement switches union and intersection.

For example, let

$$X = \{1, 2, 3, 4, 5, 6, 7\},$$

$$A = \{1, 2, 3, 4\}, \qquad B = \{3, 4, 5, 6\}.$$

Since

$$A \cap B = \{3, 4\},$$

we have

$$(A \cap B)^c = \{1, 2, 5, 6, 7\}.$$

Also,

$$A^c = \{5, 6, 7\}$$

and

$$B^c = \{1, 2, 7\}.$$

Therefore,

$$A^c \cup B^c = \{1, 2, 5, 6, 7\}.$$

Hence,

$$(A \cap B)^c = A^c \cup B^c.$$

For the other law,

$$A \cup B = \{1, 2, 3, 4, 5, 6\},$$

so

$$(A \cup B)^c = \{7\}.$$

Also,

$$A^c \cap B^c = \{5, 6, 7\} \cap \{1, 2, 7\} = \{7\}.$$

Therefore,

$$(A \cup B)^c = A^c \cap B^c.$$

Subsets and Set Equality

Two sets are equal if and only if each is a subset of the other:

$$A = B \iff A \subseteq B \text{ and } B \subseteq A.$$

This gives the **double-inclusion method**.

To prove

$$A = B,$$

we show both

$$A \subseteq B$$

and

$$B \subseteq A.$$

Some useful subset characterizations are

$$A \cap B = A \iff A \subseteq B$$

and

$$A \cap B = B \iff B \subseteq A.$$

Symmetric Difference

The symmetric difference of A and B is

$$A \triangle B = (A \setminus B) \cup (B \setminus A).$$

The symmetric difference consists of everything that is in A or B, but not both.

Using

$$A = \{1, 2, 3, 4\}$$

and

$$B = \{3, 4, 5, 6\},$$

we have

$$A \setminus B = \{1, 2\}$$

and

$$B \setminus A = \{5, 6\}.$$

Therefore,

$$A \triangle B = \{1, 2\} \cup \{5, 6\} = \{1, 2, 5, 6\}.$$

Symmetric difference can also be written as

$$A \triangle B = (A \cup B) \setminus (A \cap B).$$

In our example,

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

and

$$A \cap B = \{3, 4\}.$$

Thus,

$$(A \cup B) \setminus (A \cap B) = \{1, 2, 5, 6\}.$$

So both formulas give the same result.

Definitions:

Definition 1. *The* ***union*** *of sets A and B is*

$$A \cup B = \{x \in X : x \in A \text{ or } x \in B\}.$$

Definition 2. *The* ***intersection*** *of sets A and B is*

$$A \cap B = \{x \in X : x \in A \text{ and } x \in B\}.$$

Definition 3. *The* ***complement*** *of A is*

$$A^c = \{x \in X : x \notin A\}.$$

Definition 4. *The* ***set difference*** *of A and B is*

$$A \setminus B = \{x \in X : x \in A \text{ and } x \notin B\}.$$

Definition 5. *The* ***symmetric difference*** *of A and B is*

$$A \triangle B = (A \setminus B) \cup (B \setminus A).$$

It is the set of elements that belong to A or B, but not both.

Theorems and Proofs:

Theorem 1. *For any sets A and B,*

$$A \setminus (A \setminus B) = A \cap B.$$

Proof. We prove the equality by double inclusion.

First, suppose

$$x \in A \setminus (A \setminus B).$$

Then

$$x \in A \quad \text{and} \quad x \notin A \setminus B.$$

Since $x \in A$ but $x \notin A \setminus B$, we must have

$$x \in B.$$

Therefore,

$$x \in A \cap B.$$

Hence,

$$A \setminus (A \setminus B) \subseteq A \cap B.$$

For the reverse inclusion, suppose

$$x \in A \cap B.$$

Then

$$x \in A \quad \text{and} \quad x \in B.$$

Since $x \in B$, x cannot belong to $A \setminus B$. Thus,

$$x \notin A \setminus B.$$

Since $x \in A$ and $x \notin A \setminus B$,

$$x \in A \setminus (A \setminus B).$$

Therefore,

$$A \cap B \subseteq A \setminus (A \setminus B).$$

Since both inclusions hold,

$$A \setminus (A \setminus B) = A \cap B.$$

□

For example, if
$$A = \{1, 2, 3, 4\} \quad \text{and} \quad B = \{3, 4, 5, 6\},$$
then
$$A \setminus B = \{1, 2\}.$$

Therefore,
$$A \setminus (A \setminus B) = \{1, 2, 3, 4\} \setminus \{1, 2\} = \{3, 4\}.$$

Since
$$A \cap B = \{3, 4\},$$
the identity is verified in this example.

Theorem 2. *For any sets A and B,*
$$A \triangle B = \varnothing \iff A = B.$$

Proof. First suppose
$$A = B.$$

Since $A = B$, there are no elements of A that are not in B. Therefore,
$$A \setminus B = \varnothing.$$

Similarly,
$$B \setminus A = \varnothing.$$

Hence,
$$A \triangle B = (A \setminus B) \cup (B \setminus A) = \varnothing \cup \varnothing = \varnothing.$$

For the other direction, suppose
$$A \triangle B = \varnothing.$$

Then
$$(A \setminus B) \cup (B \setminus A) = \varnothing.$$

Therefore,
$$A \setminus B = \varnothing$$
and
$$B \setminus A = \varnothing.$$

Since
$$A \setminus B = \varnothing,$$
every element of A is also an element of B. Thus,
$$A \subseteq B.$$

Similarly,
$$B \setminus A = \varnothing$$

implies

$$B \subseteq A.$$

Therefore,

$$A = B.$$

Hence,

$$A \triangle B = \varnothing \iff A = B.$$

$\square$

For example, suppose

$$A = B = \{1, 2, 3\}.$$

Then

$$A \setminus B = \varnothing$$

and

$$B \setminus A = \varnothing.$$

Therefore,

$$A \triangle B = \varnothing \cup \varnothing = \varnothing.$$

Theorem 3. *For any sets A and B,*

$$A \cap B \subseteq A \cup B.$$

For example, if

$$A = \{1, 2, 3, 4\} \quad \text{and} \quad B = \{3, 4, 5, 6\},$$

then

$$A \cap B = \{3, 4\}$$

and

$$A \cup B = \{1, 2, 3, 4, 5, 6\}.$$

Clearly,

$$\{3, 4\} \subseteq \{1, 2, 3, 4, 5, 6\}.$$

Theorem 4. *For any sets A and B,*

$$(A \setminus B) \cap (B \setminus A) = \varnothing.$$

For our example,

$$A \setminus B = \{1, 2\}$$

and

$$B \setminus A = \{5, 6\}.$$

Thus,

$$(A \setminus B) \cap (B \setminus A) = \{1, 2\} \cap \{5, 6\} = \varnothing.$$

This makes sense because an element cannot simultaneously be in A but not B and in B but not A.

Important Formulas from Class

$$A \setminus B = A \cap B^c$$

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$

$$A \setminus (A \setminus B) = A \cap B$$

$$A \triangle B = (A \setminus B) \cup (B \setminus A)$$

$$A \triangle B = (A \cup B) \setminus (A \cap B)$$

$$A \triangle B = \varnothing \iff A = B$$

$$A \cap B \subseteq A \cup B$$

$$(A \setminus B) \cap (B \setminus A) = \varnothing$$

Questions raised in class:

- How can double inclusion be used to prove that two sets are equal?
- Why does $A \setminus B = \varnothing$ imply that $A \subseteq B$?
- Why does symmetric difference represent the elements that are in one set but not both?
- How are the two formulas

  $$A \triangle B = (A \setminus B) \cup (B \setminus A)$$

  and

  $$A \triangle B = (A \cup B) \setminus (A \cap B)$$

  equivalent?
- Why does

  $$A \triangle B = \varnothing$$

  imply that A and B must be equal?