MATH 267 Fall 2026 - Class Notes

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Summary: Today we began class with a warm up on quantifiers and notation. We discussed set builder notation as well as the quantifiers $\forall$ and $\exists$. We then talked about and looked at examples of negating quantified statements.

Notes:

- Summary of set builder notation: $\{x \in S : p(x)\}$, where S = set and p(x) = condition(s) to be satisfied, and the first part, $x \in S$, is where the elements live.
- $\exists$ means "There exists"
- $\forall$ means "For all"
- $\{a, b\} = \{b, a\}$
- $(a, b) \neq (b, a)$.
- When negating quantified statements, $\forall$ becomes $\exists$ and vice versa. For example, $\neg(\forall x \in S, p(x)) = (\exists x \in S, \neg p(x))$.

Definitions:

Definition 1. *We defined $n > 1, n \in \mathbb{N}$ as* ***reducible*** *if there exists $a, b \in \mathbb{N}, a > 1, b > 1$ such that $n = ab$. This can be stated in quantifiers by: $\exists a \in \mathbb{N}, \exists b \in \mathbb{N}(n = ab \wedge a > 1 \wedge b > 1)$.*

Definition 2. *An integer $n > 1$ is* ***prime*** *if $\forall a \in \mathbb{Z}, \forall b \in \mathbb{Z}(n \mid ab \implies (n \mid a \vee n \mid b))$.*

Definition 3. *An integer $n > 1$ is* ***irreducible*** *if it is not reducible, that is, if $\forall a \in \mathbb{N}, \forall b \in \mathbb{N}(n = ab \implies (a = 1 \vee b = 1))$.*

Theorems and Proofs:

Theorem 1. *If n is an integer n is irreducible if and only if n is prime. This is not true in general if integers is replaced by more complicated sets ("groups").*

Questions raised in class: One question raised in class today was when do you put a comma? We concluded that it really doesn't matter if you put a comma or not. For example, $\forall x \in \mathbb{R}\ x^2 \geq 0 \iff \forall x \in \mathbb{R}, x^2 \geq 0$.