MATH 267 Fall 2026 - Class Notes

8/31/2026

Jenna Klein

Summary: We started with a warm-up that discussed statements and their converses and contrapositives. We then reviewed the relationships between statements, contrapositives, and converses, and then took a quiz on these concepts. We then looked at examples where it's more useful to prove the contrapositive of a statement rather than the original statement.

Notes:

Warm-up Questions Discussed: We discussed statements and their converses and contrapositives, and then determined if the original statements and the converse statements are true or false.

1. If x is an even integer, then x^2 is an even integer. We determined that the converse was If x^2 is an even integer, then x is an even integer, and the contrapositive is if x^2 is not an even integer, then x is not an even integer. We determined that the original statement was true, but the converse was false. A counter example of this would be $x^2 = 8$.

Question: How are statements related to their contrapositives? What about their converses?

- A statement and its contrapositive are equivalent - both are true, or both are false.
- However, the truth table for the converse is NOT equivalent

P	Q	$P \to Q$	$\neg Q \to \neg P$	$Q \to P$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Question: Why would we want the converse of a statement if it's not equivalent to the original statement?

- One way to prove that two statements are equivalent is to prove both the statement and its converse are equivalent.
- $((P \to Q) \wedge (Q \to P)) \iff (P \iff Q)$

Question: When should we use the contrapositive?

- If you are struggling to prove a statement, sometimes proving the contrapositive will make more sense.

- **Example:** Prove that if n is an integer and 4 does not divide n^2, then n is odd.
- If we try to prove that $P \to Q$ directly, it might be a little tricky.
- P is stating that: $4 \nmid n^2$ and Q is stating that: n is odd, $2 \nmid n$, $n = 2k+1$ (all equivalent statements for statement Q).
- How can we unpack $4 \nmid n^2$? - Look at the negated statement: $4 \mid n^2$ which means there exists q such that $n^2 = 4q$.
- P is stating there does not exist an q with $n^2 = 4a = q$. However, arguing from "there does not exist" is hard.
- Let's try to prove the contrapositive $\neg Q \to \neg P$
- (n is even) $\to$ (4 divides n^2)]
- If n is even, there exists an integer k such that $n = 2k$. Then $n^2 = (2k)^2 = 2^2 * k^2 = 4 * k^2$. Since k^2 is an integer, we find by the definition of divides that $4 \mid n^2$

Try this: If a and b are integers and $a * b$ is off, then a is odd.

- P is stating that "a and b are integers and $a * b$ is odd" and Q is stating that "a is odd". We are tying to prove that $P \to Q$.

- **Direct Proof Attempt:** If ab is odd, then $ab = 2k + 1$ for some integer k. Our goal would be to prove that a is odd, meaning that $a = 2l + 1$.
- **Important note:** Be careful anytime you use the same letter. Only use the same letter for things you know are equal.
- We can try multiplying through by b, which gives us, $ab = 2lb + b$. We would then need to prove that $2k + 1 = 2lb + b$, except this doesn't go anywhere.
- We can also try $a = (2k + 1)/b$ but this also doesn't go anywhere. Where do we go?

- **Contrapositive $\neg Q \to \neg P$:** If a is even then $a * b$ is even.
- If a is even, then there exists an integer k such that $a = 2k$.

- Then $a * b = b * (2k) = 2bk = 2l$ where $l = kb$ is an integer
- By the definition of even, $a * b$ is even.

Definitions:

Definition 1. ***Converse:*** *The converse of the implication* $P \to Q$ *is* $Q \to P$

Definition 2. ***Contrapositive:*** *The contrapositive of the implication* $P \to Q$ *is* $\neg Q \to \neg P$

Questions raised in class: We did not have any unresolved questions or any unsolved problems today.