MATH 267: INTRODUCTION TO ABSTRACT MATH
LECTURE NOTES - AUGUST 26, 2026

DR. MCNEW

Reading: Chapter 2 of Gorkin & Daepp. **Notes:** §2.

Goals for today. By the end of class you should be able to:
- decide whether a given sentence is a statement;
- build a truth table for a compound statement form;
- explain why $P \to Q$ is *true* whenever P is false;
- negate a compound statement correctly, including an implication.

Plan (110 minutes). Reflection harvest (10) · Activity 1 (15) · Connectives (25) · Activity 2 (15) · De Morgan (20) · Activity 3 (20) · Wrap (5).

1. Reflection Harvest (10 min)

The first reading reflections came in last night. Open class by answering two or three of the *Wonder* questions on the board, crediting them by name.

This costs ten minutes and buys two things: students see that the reflections are actually read, and the questions themselves tell you what to slow down on today. Make this a standing ritual — it is the mechanism that makes assigned reading real.

2. Activity 1: Is It a Statement? (15 min)

Definition 1. A **statement** is a sentence that is true or false, but not both.

Activity 1. For each sentence, decide whether it is a statement. If it is, decide whether it is true or false.

(a) $2 + 2 = 5$
(b) $2 + 3 \leq 5$
(c) 7 is a prime number.
(d) $x + 6 = 0$
(e) All prime numbers are odd.
(f) 2 is an odd prime number.
(g) This sentence is false.

Instructor notes.
- (a) statement, false. (b) statement, true — note $\leq$ holds because $2 + 3 = 5$; some students read $\leq$ as strict.
- (c) statement, true. (e) statement, false (2 is prime and even). (f) statement, false.
- (d) is *not* a statement: its truth depends on x. This is the important one — it is a *predicate*, and it becomes a statement only once x is pinned down or quantified. Flag that almost every interesting sentence in mathematics is of this kind, which is why quantifiers arrive next week.
- (g) is not a statement: assuming it true makes it false and vice versa. Worth 90 seconds, not more. Mention that Chapter 6's Spotlight on paradoxes returns to this.

- Small trap in (c) and (f): we have not defined "prime" yet — we defined *irreducible* on Monday. Let students notice. If nobody does, point it out.

3. Connectives and Truth Tables (25 min)

Introduce, with a truth table for each: negation $\neg P$, conjunction $P \wedge Q$, disjunction $P \vee Q$, implication $P \to Q$, biconditional $P \leftrightarrow Q$.

Two points deserve real time.

"Or" is inclusive. $P \vee Q$ is true when both hold. Everyday English usually means the exclusive version ("soup or salad"), so the convention has to be stated and then used relentlessly.

A false hypothesis makes an implication true. This is the hardest row of the semester:

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

Students want $P \to Q$ to be false, or meaningless, when P is false. Offer the promise version: "if you score a 95, I'll give you an A." If you never score a 95, I have not broken the promise no matter what grade you get. The only way to break the promise — the only F row — is to score the 95 and not get the A. Say the phrase *vacuously true* and write it down; it will recur all term.

4. Activity 2: Verifying an Equivalence (15 min)

Two statement forms are **equivalent** when they have identical truth tables.

Activity 2. At the board, in groups: build a single truth table with columns for P, Q, $P \to Q$, $\neg(P \to Q)$, $\neg Q$, and $P \wedge \neg Q$. What does the result tell you?

Instructor notes. The columns for $\neg(P \to Q)$ and $P \wedge \neg Q$ agree in all four rows, establishing

$$\neg(P \to Q) \leftrightarrow (P \wedge \neg Q).$$

Draw the moral in words before moving on, because it is the one they will actually use: *the negation of "if P then Q" is not another implication.* It is a single claim asserting that P happens and Q fails. Students reflexively negate an implication into "if P then not Q", and this table is the cure.

5. De Morgan's Laws (20 min)

Theorem 1. If P and Q are statements, then
(1) $\neg(\neg P) \leftrightarrow P$;
(2) $\neg(P \wedge Q) \leftrightarrow (\neg P \vee \neg Q)$;
(3) $\neg(P \vee Q) \leftrightarrow (\neg P \wedge \neg Q)$;
(4) $\neg(P \to Q) \leftrightarrow (P \wedge \neg Q)$.

Part (4) is Activity 2. Prove (2) at the board as the model, then assign (1) and (3) to be finished on paper — they are quick, and doing one alone is worth more than watching three.

Emphasize the pattern in (2) and (3): negation passes through the connective and *flips it.* "Not (both)" becomes "at least one fails"; "not (either)" becomes "both fail." Next week the same flipping rule appears for quantifiers, where $\forall$ and $\exists$ trade places.

6. Activity 3: True or False, Then Negate (20 min)

Activity 3. For each statement, decide whether it is true or false, and then state its negation in plain English.

(a) If $5 \mid 60$, then $5 \mid 6$ or $5 \mid 10$.
(b) If $5 \mid 99$, then $5 \mid 9$ or $5 \mid 11$.
(c) If $8 \mid 24$, then $8 \mid 6$ or $8 \mid 4$.
(d) If $8 \mid 48$, then $8 \mid 24$ or $8 \mid 2$.

Instructor notes. These combine Monday's divisibility definition with today's truth table, and each one traps a different reflex.

- (a) **True**. Hypothesis $5 \mid 60$ is true; the conclusion is a disjunction and $5 \mid 10$ holds, so it is true. Students who insist both disjuncts must hold are reading $\vee$ as "and" — catch this.
- (b) **True**, vacuously: $5 \nmid 99$. The conclusion is false, and it does not matter. This is the one most of the class will get wrong. Make them articulate why before you confirm it.
- (c) **False**. $8 \mid 24$ is true, but $8 \nmid 6$ and $8 \nmid 4$, so the conclusion fails. This is the only F row — true hypothesis, false conclusion.
- (d) **True**: $8 \mid 48$ and $8 \mid 24$.
- Negations all follow the shape from Activity 2. For (c): "8 divides 24, and 8 divides neither 6 nor 4." Note this negation is a *true* statement, consistent with (c) being false. That correspondence is worth pointing out explicitly.

7. Wrap-Up (5 min)

Preview Monday: Chapter 3 takes the implication $P \to Q$ apart into its converse and its contrapositive — one of which is equivalent to the original and one of which is emphatically not. Today's truth tables are the tool that settles which.

8. Assigned for Monday

- Read Chapter 3 (Kumchev §2).
- Reading reflection due Sunday 8/30 at 11:59pm.
- Finish parts (1) and (3) of De Morgan's laws by truth table.