MATH 267 Fall 2026 - Class Notes

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Summary:
Today we looked at the limit of sequences and whether they converge. We went over the proof strategy for determining convergence and what it means if it does not converge.

Notes:

Warm up:
$x_n = (4n+1)/(2n-3)$: Limit=2
How large do we have to make N before $|x_n - 2| < 0.1$? $n > 36$
What about $\epsilon = 0.01$. Then $|x_n - 2| < 0.01$ When $n > 352$

- ϵ measures how far a term is from the limit

Proof Strategy for convergence:

1. Guess the value of the limit L using techniques from Calculus
2. Simplify the distance $|x_n - L|$ algebraically
3. Find a simpler upper bound for it (Replace expressions with bigger quantities
4. Set it equal to epsilon ans solve for n to get a lower bound **(end of scratch work)**
5. Write down a proof that for all $n > N, |x_n - L| < \epsilon$

Definitions:

Definition 1. *We say a sequence $\{x_n\}$* ***converges*** *to L if, for any $\epsilon > 0$, there exists an N such that for all $n > N$, $|x_n - L| < \epsilon$.*

Definition 2. *If our sequence does not converge to a limit L we say that the sequence* ***diverges****.*

Theorems and Proofs:

Prove that $x_n = (5n-2)/(2n+3)$ converges to 5/2

Proof. Fix $\epsilon > 0$. We will show that $|x_n - \frac{5}{2}| < \epsilon$ for all $n > N$. Set N=$\frac{5}{\epsilon}$. $|x_n - \frac{5}{2}| = |\frac{5n-2}{2n+3} - \frac{5}{2}| = |\frac{2(5n-2)}{2(2n+3)} - \frac{5(2n+3)}{2(2n+3)}| = |\frac{10n-4-10n-15}{2(2n+3)}| = |\frac{-19}{(4n+6)}| = \frac{19}{(4n+6)} < \frac{19}{4n}$. Now for every $n > \frac{5}{\epsilon}, |x_n - \frac{5}{2}| < \frac{19}{4n} < \frac{19}{4(\frac{5}{\epsilon})} = \frac{19\epsilon}{20} < \epsilon$ □

Prove that $x_n = 2(-1)^n$ diverges

Proof. Suppose for contradiction that x_n converges to some L. Pick $\epsilon = 1$. Then there exists an N so that if $n > N$ then $|x_n - L| < \epsilon = 1$. Suppose $L \geq 0$. Pick an odd $n > N$. Then $|x_n - L| = |-2 - L| \geq |-2| = 2$. This contradicts that $|x_n - L| < 1$ for all $n > N$. Now suppose $L < 0$. Pick an even $n > N$. Then $|x_n - L| = |2 - L| > 2$ since $L < 0$. This also contradicts that $\epsilon = 1$ □

Questions raised in class:

For any $\epsilon > 0$ can we guarantee that our sequence is eventually always closer to the limit?: If so then then limit exists, if not then this is not the right limit or the sequence diverges.
We ended class with this problem: Find a limit for $x_n = \frac{2n^2+3}{5n^2-n+4}$. We assumed L=$\frac{2}{5}$. We need to find N so that $|x_n - 2/5| < \epsilon$.