## Activity Day 4: Descriptive Statistics

## Purpose:

1. Students will collect experimental data to construct graphical and algebraic representations for the relationship between two variables (Barbie Bungee Jumping number of rubber bands in bungee cord and minimum safe drop distance; the Urban Heat Island - temperature and albedo);
2. Students will use the graphical and algebraic relationships that they have found to predict a value of the dependent variable given a value of the independent variable;
3. to investigate the meaning of the mean and median

## Description:

During this activity students will use tools from data analysis that correspond to Maryland's High School Voluntary State Curriculum for Algebra/Data Analysis. Topics will include construction and examination of scatterplots that examine linear trends, and making predictions based upon those plots.

Students will work in teams of three to design a bungee cord that optimizes the thrill of bungee jumping without causing bodily harm to the jumper. Each team will use a Barbie (or Ken) doll to represent the bungee jumper and a series of rubber bands to simulate the cord. The teams will collect data by varying the length of the cord and recording the resulting minimum distance of a safe jump. Each team will enter its data into lists on a graphing calculator, and construct a scatterplot to discover the linear relationship between the length of Barbie's bungee cord (number of rubber bands) and the minimum safe distance of a jump. Students will eye-ball fit the data with a line, and use the calculator's "line of best fit" to graphically describe the linear relationship between cord length and jump distance. Teams will use this relationship to predict the number of rubber bands necessary for an exciting, but safe, two-story jump for Barbie / Ken.

Students will strengthen their understanding of linear regression concepts as they revisit the Urban Heat Island Phenomena from Activity Day 1. Students will construct a scatterplot for albedo (reflectivity of light) and temperature, discovering a confounding third variable of moisture. Using an "eyeball" fit students will graphically describe this relationship, make predictions, and discuss limitations of the relationship.

The final activity will involve an exploration of measures of center. Students will be introduced to the concepts of mean, median, and mode through several hands-on activities. Students will be able to visualize the mean as both a balance point for a data set and a leveler (or fair share). After the hands-on activities, students will examine a histogram of the distribution of electoral votes to Obama and McCain. Students will locate both the mean and median on this graph, and then compare frequencies of electoral votes for states leaning to each candidate.

Breakfast and lunch will be provided by the grant.

## Materials:

General Materials:

- Breakfast
- Lunch
- Reflection journals
- Towson University student binders
- Name tags \& markers
- Camera
- Paper goods

Barbie Bungee Jumping Materials:

- 10 Barbie and Ken dolls
- One dozen meter sticks
- \#32 rubber bands
- Handouts for Barbie Bungee Jumping
- Class set of TI-84 calculators
- Graph paper
- 14 rulers
- Tape measure

Urban Heat Island Phenomena Materials:

- Graph paper
- Colored pencils
- 7 Rulers
- Light meter
- IR thermometer
- Control board for albedo measurements
- 2 Red spirit thermometers
- 'Standards' board: 3 albedo chips; 2 paper towels
- Spray bottle (water)
- Laptop computer with statistical package

Measures of Center Materials:

- 1 inch cubes
- Post-it notes
- Smarty candies
- Electoral maps
- Histogram of electoral votes
- Excel list of number of electoral votes by state


## Staffing:

- Cooper (lead), Tomayko, and Roberge;
- Bello, two FHHS teachers;


## General Schedule:

8:30 meet Ameerah at FHHS; set up food; check on Gym availability for test jumps
8:45 breakfast with students
9:00 prizes for estimating circumference of the Earth; description of the day's activities
9:15 Barbie Bungee Jumping
10:45 Urban Heat Island
11:00 Measures of Central Tendency and Journal Writing
12:30 lunch
1:00 end of day

## Detailed Schedule \& Activity Description - following pages:

## Barbie Bungee and Kamikaze Ken

Team Member Names: $\qquad$


Problem:
Your team has been hired to work for the Daredevil Entertainment Company TM. This company provides rock climbing, sky diving, "extreme skiing", and cliff diving adventures to the public. To keep up with market demand, the company's board of directors decided to add bungee jumping to its list of offerings. Your task is to simulate the testing of the drop height for a bungee cord that optimizes the thrill of jumping without actually hitting the floor.


Step 1: Collecting Data

1. Create a double-loop to wrap around Barbie's feet. A double-loop is made by securing one rubber band to another with a slip knot.
2. Wrap one end of the double-loop tightly around Barbie's feet.
3. Select an initial drop height (in centimeters) and use a piece of tape to mark the jump line.
4. Hold the end of the rubber band at the jump line with one hand and drop Barbie from the line with the other hand.
5. Have a partner keep track of the lowest point Barbie reaches. If the doll hits the ground, increase the jump height. Conduct 3 trials and record the distances (in centimeters) in the table. Average the distances and record that value in the table.
6. Continue adding rubber bands to complete the table. Note: You are skipping collecting data for 4 rubber bands.

| Number of <br> Rubber Bands | Maximum Height You Can Safely Jump |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Trial 1 | Trial 2 | Trial 3 | Average |
|  |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |

Step 2: Modeling the Data

1. Using a graphing calculator, enter the number of rubber bands data in L1 and the average heights in L2. (Press STAT 1 under the EDIT menu to edit your list. For any list that contains old data, use your arrow key to highlight the list name and press

## CLEAR ENTER

Here is an example of what your calculator screen might look like.

| L1 | Lz | L3 | 2 |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & 1 \\ & 2 \\ & 2 \\ & 5 \\ & 6 \end{aligned}$ | 31 53 73 117 187 |  |  |
| L2(6) = |  |  |  |

2. Plot your data. To get to STAT PLOT, press

## 2nd



3．Now go back to STAT PLOT．Select Plot 1 and use the following settings：


4．To show your graph，press
 for ZoomStat．

## STAT

5．Do a linear regression to find the line of best fit．Press and arrow to the right to highlight CALC．Arrow down to \＃4 LinReg（ax＋b）and press enter．

Your screen should look like this：

```
Plots 0 ff
Done
LinReg（ax＋b）
```

Press enter．

6．Your screen should look similar to this（The values will be different）：

```
Liに民シヨ
    \(\cdots=8 \times 6\)
    \(9=212\)
    \(b=9.8\)
    \(\mathrm{r} 2=.9995900889\)
    \(r=.999694998\)
```

7．What is the slope of the equation？ $\qquad$
8．Interpret the value of the slope： $\qquad$
$\qquad$
9. What is the value of the $y$-intercept? $\qquad$
10. Why is the value of the $y$-intercept not equal to zero? $\qquad$
$\qquad$
11. Write the equation of the line that describes the relationship between the number of rubber bands and Barbie's minimum jump distance: $\mathbf{Y}=$ $\qquad$
12. Press $Y=$
13. Press graph to see how well your line fits the data. Use the trace button to see the values.
14. From your graph and equation, predict the jump distance for 4 rubber bands. We predict
$\qquad$ cm because $\qquad$
$\qquad$ .
15. Use the meter stick to mark your predicted jump distance for 4 rubber bands. Test your prediction. Describe your findings: $\qquad$
$\qquad$
$\qquad$
$\qquad$ .
16. From your graph and equation, predict the jump distance for 15 rubber bands. We predict $\qquad$ cm because $\qquad$
$\qquad$ .
17. Let's go to the Gym. Use the meter stick to mark your predicted jump distance on the bleachers. Test your prediction. Describe your findings: $\qquad$
$\qquad$
$\qquad$
$\qquad$ .

## Urban Heat Island Phenomena revisited through the lens of data analysis

Students will revisit the Urban Heat Island phenomena, again brainstorming ideas as to why cities are warmer than their surrounding rural areas. Working in groups of two, students will graph the data that they collected from Activity Day 1. Students will attempt to discover a relationship between albedo and temperature. This relationship will not be as clear as the linear relationship that they encountered with Barbie's length of bungee cord and minimum safe distance jump. Upon further discussion and exploration of the data, students may hypothesize that temperature may also be affected by evaporation. Students will code the points on the scatterplot according to moisture level (no moisture, some moisture). Students will then attempt to explain temperature as dependent upon both albedo and moisture level, drawing conclusions as to which factor plays the more significant role in the Urban Heat Island Phenomena. Students will relate their findings to the urban city. Why would urban areas have greater / lesser albedo levels than rural areas? Why would urban areas be more / less moist than rural areas?

## UHI Background:

- Urban areas tend to be warmer than rural areas.
- This could be explained by the low albedo of urban asphalt and dark-colored roofs:
- Low albedo (dark colors), such as asphalt, will absorb more sunlight.
- High albedo (light colors) will reflect more sunlight.
- This could be explained by the lower evaporation from urban pavement:
- Moist surfaces (like grass or trees) will use energy to evaporate and will be cool.
- Dry surfaces (like pavement) will use energy to get warmer.


## The Urban Heat Island

Group Members:
Date:
Time:__ Weather Conditions:
Air temp:___ ${ }^{\circ} \mathbf{F}$

| Description of Surface Type | Reflected light $\uparrow$ <br> $($ lux $)$ | Total Sunlight $\downarrow$ <br> $($ lux $)$ | Albedo <br> $(\%)$ | Temperature <br> $\left({ }^{( } \mathbf{F}\right)$ |
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Albedo is the amount of sunlight that an object reflects. An object that reflects half of the light that strikes it has an albedo of $50 \%$. An object that reflects all of the light has an albedo of $100 \%$. What color would a surface be if it had an albedo of $0 \%$ ?

We'll use a light meter to measure the amount of sunlight that shines on an object, and then again to measure the amount of light that reflects off of the object.

Temperature is a measure of how much energy an object has absorbed. We'll be using an Infrared Thermometer to measure the temperature of different surfaces.

## Investigating Measures of Center

Measures of Center describe the middle or center of the data set. There are three common measures of "center" - mean, median, and mode.

## Activity 1 - The Mean as a Leveler

Students will work in groups of three. Each group will be given a slip of paper noting the data set (3562153221) and 30 one inch cubes asked to construct stacks from the blocks such that each stack height is equal to one data value. Students will then investigate the meaning of mean, median, and mode in this context. Students will first be asked to find the median of the data set. Students may choose to order their blocks and then identify the median. Next students will be asked to identify the mode. A likely misconception will be to choose the stack of greatest height - data value 6. A prompt that can be used to redirect the students is "What do the stack heights represent?" The mode is the most frequent data value, in this case the most common height. Finally, students will be asked to find the mean (average) number of blocks. They will specifically be told that they can not find the mean by using the algorithm of summing all values and dividing by the number of values. Students will need to attempt to distribute the blocks into stacks of equal heights. It will be important that students keep 10 stacks as they redistribute blocks. Students may be asked to imagine the scenario that there are ten children with various numbers of candy bars (the data values 356215322 1). What is the average number of candy bars that each child has? In this context the mean is seen as a leveler or fair-share.

## Activity 2 - The Mean as Fair-Share (Leveler)

Students will combine into larger groups of approximately six. Students will be given various numbers of Smarty candy packages (such that the total for each group is equal to twice the group size. Some group members may receive no Smarty packages). Students will be asked to find the average number of Smarty packages per person within the group. Again, students will be told that they can not find the mean through addition and division. Students will redistribute the Smarty in a fair-share method to find the mean number of Smarty packages per group member.

## Activity 3 - The Mean as the Balance Point

Students will be presented with the following scenario that a group of Algebra I teachers leave for a weeklong conference in San Francisco. Each teacher brings exams to grade along the way. Some have many exams to grade, some have only a few. The teachers decide to reallocate the exams so that each teacher has the same number of exams to grade. They're hoping to find time to go as a group and explore the sites of San Francisco. Students will be given the following graph and told that it shows the number of exams that each teacher brought.


Students will work collectively as they discuss how to answer the following questions:

- What does the graph tell you about the number of exams that the teachers brought?
- What is the significance of the bar above 13 on the horizontal axis?
- What is the greatest number of exams that any teacher brought?
- What is the least number of exams that any teacher brought?
- How many teachers went on this trip?
- Find the median number of exams that teachers brought. How did you find this value? Mark the median value on your graph.
- Do you expect the mean (average) number of exams to be smaller / larger than the median? Why?

Next students will model the re-allocation of exams, conceptually developing the idea of the mean as the balance point of the graph. The initial graph is balanced on the unknown mean. Think of the graph as a see-saw, perched on a fulcrum. Students will need to find where this balance point, or fulcrum, lies. Working as a class, students will use a poster-board graph of the same data to determine the balance point. Each bar of this class graph will be comprised of Post-it notes - one for each teacher. Three teachers brought 12 exams; therefore there will be three Post-it notes above the value 12 on the horizontal axis. Two students will be asked to volunteer to help with the re-allocation of exams. The goal is to find the mean number of exams that each teacher should grade. This will be the balance point of the graph. Students will be reminded of how a see-saw can remain level if the number of children [of equal weight] each multiplied by his/her distance from the center of the board is the same for each side. [It will be important to note that the see-saw will balance if all students are directly over the center [fulcrum]. In the case of our graph, we don't know the location of the fulcrum, or balance point, of the "see-saw." The horizontal axis can be viewed as a see-saw with unknown center. Students may incorrectly perceive the center of the horizontal axis to be the balance point of the graph. Discussion will need to counter this misconception. Students will find this center value by moving Post-it notes systematically to the center. The volunteer stationed on the left side of the graph will move a Post-it Note from the extreme left toward the center. The volunteer on the right side must counter this move by moving a Post-it from the right side of the graph closer to center. If the volunteer on the left moves one Post-it one space to the right, the volunteer on the right will move one Post-it one space to the left. If the volunteer on the left chooses to move one Post-it note three steps to the left, the volunteer on the right can either move one Post-it three steps to the right, or a combination of one Post-it one step to the left and a second Post-it two steps to the left. There are many possibilities. This procedure will
continue until students have reached their goal of fair-share of the exams: All Post-it notes will rest on one value on the horizontal axis. Students will consider the formula for the mean.

Students can read the original data values as recorded on individual Post-it notes. The sum of all data values is $12+12+12+13+13+14+15+18+20+21=150$ total exams. The students will also note that if the exams are re-allocated, the total number of exams will remain the same -150 . Students will be encouraged to see that as a result of the re-allocation, each of the 10 teachers has 15 exams to grade, for a total of 150 exams.

So $12+12+12+13+13+14+15+18+20+21=10 \cdot 15$
Or, alternatively, $\frac{12+12+12+13+13+14+15+18+20+21}{10}=15$
Students have now linked the idea of the mean as fair-share to the well-known formula of the sum divided by the total. The mean is both a leveler (fair-share) and a balance point of the data set.

Discussion should return to the relative position of the mean and median. Why is the mean greater than the median? Discussion about the effect of outliers, or few extreme values, on the mean will follow.

## Activity 4 - Distribution of Electoral Votes

Students will be given a map of the USA illustrating the distribution of electoral votes by state. Students will discuss how to interpret the different colors of the states - red indicates supporting McCain, pink indicates leaning toward McCain, yellow indicates undecided, light blue indicates leaning toward Obama, and dark blue indicates supporting Obama. Students will be asked to make observations. Likely, students will notice that there is a lot of red and pink on the map for McCain compared to Obama's blue and light blue. The area of red as compared to the area of blue does not seem to correspond to Obama's position in leading the polls. Students can read the bar above the graph to see that Obama is expected to have 277 of the 538 electoral votes. To help elucidate that the number of electoral votes per state is not the same for each state, nor based upon the state's area, students will be given a black and white histogram showing the distribution of the electoral votes across the 50 states. Students will be asked to make observations about the graph, such as there are many states with few electoral votes, and only a few with many electoral votes. Students will be asked to identify the states indicated by the four largest values on the graph (California, Texas, New York, and Florida). Discussion will be encouraged about the importance of securing states with large populations (large number of electoral votes). Next students will be given another histogram, similar to the first, with the added feature that each bar has been divided into categories that show support for the candidates (safe Obama, leaning Obama, undecided, leaning McCain, and safe McCain). Students will be asked to observe which candidate has secured a greater number of states with "many electoral votes." Students will be asked to find the mean number of electoral votes (the fair-share value if each state had equal input into selecting the President) and the median
(typical number of electoral votes for a state) on this graph. Students will be asked to predict (and justify) which is larger, the mean or median? Students will conclude by explaining how a democratic candidate can win the election even if the map of the USA is a sea of red. A candidate needs only to secure 270 votes to win the Presidency. Students will be asked to find and name the minimum number of states needed to win (California, Texas, New York, ....).

## Reflections

Students will be asked to choose and respond to a prompt to reflect about their experiences from either today's activities or Activity Day 3.

