Reading: Chapter 3, Sections 3.4

H29. Find the leading behavior of both solutions of $y'' = \sqrt{x}y$ as $x \to \infty$.

H30. For each of the following, be sure to justify your answer.
   (a) For what values of $k$ is $x^k \ll e^{1/x}$ as $x \to \infty$?
   (b) For what values of $k$ is $e^{1/x} \ll x^k$ as $x \to \infty$?
   (c) For what values of $k$ is $x^k \ll e^{1/x}$ as $x \to 0^+$?
   (d) For what values of $k$ is $e^{1/x} \ll x^k$ as $x \to 0^+$?

H31. Find the leading behavior of both solutions to $y'' = e^{-3/x}y$ as $x \to \infty$.

   Hint: Our usual approach using the method of dominant balance will yield differential asymptotic relations containing the term $e^{-3/x}$ which can’t be integrated in elementary functions. Because this term appears in a differential asymptotic relation, consider proceeding by replacing difficult terms involving $e^{-3/x}$ by simpler functions $f(x)$ that are asymptotic to your difficult term.

H32. Let $L(x)$ be the leading behavior to the solution that you found in H29. Look for a correction in the form $y(x) = L(x)w(x)$
   (a) Find a differential equation for $w(x)$.
   (b) Write $w(x) = 1 + \epsilon(x)$ for $\epsilon \ll 1$ as $x \to \infty$, and obtain an estimate for $\epsilon$ via the method of dominant balance.
   (c) Find a second term in your correction, again via dominant balance.

H33. Find three terms in the local behavior as $x \to 0^+$ of a particular solution to $x^3y'' + y = x^{-4}$.

H34. Find an asymptotic power series expansion as $x \to 0^+$ for $f_1(x) = \frac{1}{1+x}$. Do the same for $f_2(x) = e^{-1/x} + \frac{1}{1+x}$. Compare.