Reading: Chapter 6, Sections 6.1 - 6.3

61. Consider the integral

\[ I(x) = \int_x^1 \cos(x t) \, dt \]

(a) This is sufficiently simple that the integral can be evaluated directly. Do so, and express the result as a series in powers of \( x \). Retain only the first few terms.

(b) Apply 6.2.1 to find the leading behavior of \( I(x) \). Verify it agrees with the result above.

62. Consider the integral

\[ I(x) = \int_x^1 \sin(x t) \, dt \]

(a) This is sufficiently simple that the integral can be evaluated directly. Do so, and express the result as a series in powers of \( x \). Retain only the first few terms.

(b) Apply 6.2.1 to find the leading behavior of \( I(x) \). Verify it agrees with the result above. Unlike the previous problem however, some creativity is required to construct an appropriate choice for \( f_0(t) \).

63. Find three nonzero terms in the local behavior of

\[ \int_0^1 \sqrt{1 + x^3 t^3} \, dt \]

as \( x \to 0^+ \). [Hint: The Binomial Theorem.]

64. Show

\[ \int_0^x t^{-1/2} e^{-t} \, dt \sim \frac{e^{-x}}{\sqrt{x}} \left[ 1 + \sum_{n=1}^{\infty} \frac{(-1)^n (2n)!}{2^{2n} n!} x^{-n} \right] \]

as \( x \to \infty \). [Remark: The issue here is proving the result is an asymptotic series, not calculating the terms!]

65. Consider the behavior as \( x \to \infty \) of

\[ \int_1^2 e^{x \cosh t} \, dt \]

(a) Write this integral in the form 6.3.17, and identify \( a, b, f(t), \) and \( \phi(t) \).

(b) Is it the case that \( \phi(t), \phi'(t), \) and \( f(t) \) are continuous?

(c) Is it the case that \( \phi'(t) \neq 0 \) for all \( a \leq t \leq b \)? Is is the case that \( f(a) \neq 0 \) and \( f(b) \neq 0 \).

(d) Apply 6.3.19 to obtain an asymptotic expression for the integral as \( x \to \infty \).

(e) Explain any differences between your answer and the expression in Example 6, p. 259.