1. Identify all of the singular points for
   (a) \( y' - y/2 = 0 \),
   (b) \( y' - y/(2x) = 0 \),
   (c) \( y' - y/(2x^2) = 0 \).

2. How do you use the method of Frobenius to find a single solution of a differential equation?

3. How do you use the method of Frobenius to find a second solution of a differential equation when the indicial equation has repeated roots?

4. For two functions \( f(x) \) and \( g(x) \), what does it mean to say that \( f(x) \sim g(x) \) as \( x \to x_0 \)?

5. For two functions \( f(x) \) and \( g(x) \), what does it mean to say that \( f(x) \ll g(x) \) as \( x \to x_0 \)?

6. Use the Liouville substitution to find the controlling factors for the solution of \( x^3y'' = y \) near \( x = 0^+ \).

7. Suppose that \( \frac{3}{2}x^{-5/2} + C'' - 2x^{-3/2}C' + (C')^2 = 0 \) for \( C(x) \ll 2x^{-1/2} \) as \( x \to 0^+ \). Use the method of dominant balance to determine \( C \) (asymptotically).

8. Given a function \( f(x) \), what does it mean to say that \( f(x) \sim \sum_{n=0}^{\infty} a_n(x - x_0)^n \) as \( x \to x_0 \)?

9. Suppose that
   \[
   y(x) \sim \frac{e^x}{\sqrt{2\pi x}} \left[ 1 - \frac{(100 - 1)}{1!(8x)} + \frac{(100 - 1)(100 - 9)}{2!(8x)^2} + \frac{(100 - 1)(100 - 9)(100 - 25)}{3!(8x)^3} + \cdots \right]
   \]
   as \( x \to \infty \). What can you say about the error in using the approximation
   \[
   y(x) \approx \frac{e^x}{\sqrt{2\pi x}} \left[ 1 - \frac{(100 - 1)}{1!(8x)} \right]
   \]

10. Suppose that \( f(x) \sim \sum_{n=0}^{\infty} a_n(x - x_0)^n \) as \( x \to x_0 \).
    
    (a) Find the asymptotic series for \( \int_{x_0}^{x} f(t) \, dt \), and indicate precisely what assumptions are needed to ensure its validity.
    
    (b) Find the asymptotic series for \( f'(x) \), and indicate precisely what assumptions are needed to ensure its validity.


12. Use a perturbation series to solve \( x^3 - (4 + \epsilon)x + 2\epsilon = 0 \) up to \( O(\epsilon^2) \).

13. Consider the boundary-value problem \( \epsilon y'' + 2y' + 2y = 0 \), \( y(0) = 0 \), \( y(1) = 1 \). Find the outer solution to \( O(1) \).

14. For the previous problem, assume that there is a boundary layer at \( x = 0 \), and find the inner solution to \( O(1) \).

15. How do you use an intermediate variable to match an inner and an outer solution?