**Math 674**  
**Assignment #9**  
**Due October 31, 2012**

**Reading:** 2.5, 2.6

1. 2.4.1.a

2. 2.4.1.b [Warning: I found some typos in the problem in my printing of the book.]

3. 2.4.2. Recall that the principal moments of inertia $A, B, C$ are the eigenvalues of the tensor

$$I = \iiint_V \rho(x,y,z) \begin{bmatrix} y^2 + z^2 & -xy & -xz \\ -xy & x^2 + z^2 & -yz \\ -xz & -yz & x^2 + y^2 \end{bmatrix} \, dx \, dy \, dz.$$

- Recall that all symmetric matrices are diagonalizable, meaning that if $A$ is symmetric, there is a diagonal matrix $D$ and an invertible matrix $P$ so that $A = P^{-1}DP$.
- Note also that for any pair of matrices, $\text{Tr}(AB) = \text{Tr}(BA)$. You should be able to prove this!
- Note also that if $A$ is symmetric, then $\text{Tr}(A)$ is the sum of the eigenvalues of $A$. You should be able to prove this!

4. Prove that

$$E(r^2) = \int_{0}^{\pi/2} \sqrt{1 - r^2 \sin^2 \phi} \, d\phi = \frac{\pi}{2} \left[ 1 - \left( \frac{1}{2} \right)^2 \frac{r^2}{1} - \left( \frac{1}{2} \cdot \frac{3}{4} \right)^2 \frac{r^4}{3} - \left( \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \right)^2 \frac{r^6}{5} - \cdots \right]$$

- Hint. Let $I_n = \int_{0}^{\pi/2} \sin^n \phi \, d\phi$. Prove $(n+2)I_{n+2} = (n+1)I_n$.
- Use the binomial theorem.

5. 2.4.6.