Math 674
Assignment #6
Due October 10, 2012

Read: 2.3 (Pages 74-82.)

1. Use conformal mappings to solve the problem $\triangle u = 0$ in the first quadrant, with $u = 0$ on $y = 0$ and $u = 1$ on $x = 0$.

   $u = 1$
   $\Delta u = 0$
   $u = 0$

   Hint: Consider $\zeta = \ln z$.

2. Use conformal mappings to solve the problem $\triangle u = 0$ in the sector $0 \leq \theta \leq \theta_0$, with $u = 0$ on $\theta = 0$ and $u = u_0$ on $\theta = \theta_0$.

   $u = u_0$
   $\Delta u = 0$
   $u = 0$

3. Use conformal mappings to solve the problem $\triangle u = 0$ in the first quadrant with $u = 0$ on $x = 0$, with $\frac{\partial u}{\partial y} = 0$ on $y = 0$ for $0 < x < 1$, and $u = 1$ on $y = 0$ for $x > 1$.

   $u = 0$
   $\Delta u = 0$

   Hints: Use the transformation $z = \sin \zeta$. To write $\zeta = \xi + i\eta$ in terms of $z = x + iy$, first prove that

   $$\frac{x^2}{\sin^2 \xi} - \frac{y^2}{\cos^2 \xi} = 1.$$
This is a hyperbola in the $xy$ plane with foci at $(\pm 1,0)$. Recall the geometric property of hyperbolas that the difference of the distances from a point on the hyperbola to the foci is twice the distance between the vertices.

4. Use conformal mappings to solve the problem $\Delta u = 0$ in the region $0 \leq x \leq \pi/2$, $y > 0$ with the conditions $u = 0$ for $y = 0$ or $x = \pi/2$, and $u = 1$ for $x = 0$.

5. Use conformal mappings to solve the problem $\Delta u = 0$ in the semicircular region $0 \leq r \leq 1$, $0 \leq \theta \leq \pi$ where we have the boundary conditions $u = 0$ for $r = 1$ and for $\theta = \pi$, while $u = 1$ for $\theta = 0$.

6. 2.2.1.a
7. 2.2.1.b
8. 2.2.1.c