Math 673
Assignment #11
Applications of the Laplace Transform
Due April 20, 2005

Name______________________________

1. Solve the problem
\[
\begin{align*}
y'' + 6y' + 9y &= \sin t \\
y(0) &= 1 \\
y'(0) &= 0
\end{align*}
\]
by taking the Laplace transforms and inverting.

2. Solve the problem
\[
\begin{align*}
y'' + y' - 2y &= 0 \\
y(0) &= 1 \\
\lim_{t \to \infty} y(t) &= 0
\end{align*}
\]
Hint: Let \( y'(0) = c \) and solve the equation together with the first boundary condition. Then use the solution so obtained and the second condition to determine the value of \( c \).

3. Prove that
\[
\frac{d}{dx} \{ x^n J_n(ax) \} = ax^n J_{n-1}(ax),
\]
\[
\frac{d}{dx} \{ x^n I_n(ax) \} = ax^n I_{n-1}(ax).
\]

4. Prove that
\[
\mathcal{L} \{ t J_1(at) \} = \frac{a}{(s^2 + a^2)^{3/2}},
\]
\[
\mathcal{L} \{ t I_1(at) \} = \frac{a}{(s^2 - a^2)^{3/2}}.
\]
Hint: Use the previous problem and properties of the Laplace transform.

5. Use Laplace transform techniques to solve
\[
\begin{align*}
xy'' - y' - xy &= 0 \\
y(0) &= 0.
\end{align*}
\]

6. Solve
\[
\begin{align*}
x'' - x + y' - y &= 0 \\
-2x' - 2x + y'' - y &= e^{-t} \\
x(0) &= 0 & x'(0) &= -1 \\
y(0) &= 1 & y'(0) &= 1.
\end{align*}
\]