Math 667
Homework Questions #8

Remark: Each answer will be graded on the basis of its correctness, its logical structure, and how well it is written (style, form, spelling and grammar).

34. Let
   \[ M_\theta = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \quad \text{and} \quad M_\psi = \begin{pmatrix} \cos \psi & -\sin \psi \\ \sin \psi & \cos \psi \end{pmatrix}. \]
   What is the matrix product \( M_\theta M_\psi \)? Explain your result geometrically.

35. What is the geometric interpretation of the transformation of the plane given by the matrix
   \[ \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \]?
   Suggestion: Apply the matrix to the vertices of the unit square.

36. Calculate
   \[ \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^n \]
   for \( n = 2, 3, 4, 5 \). What is the geometric effect of each of these transformations?
   What occurs in the general case? That is, calculate
   \[ \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^n \]
   for arbitrary \( n \), and explain its geometric significance.