**Remark:** Each answer will be graded on the basis of its correctness, its logical structure, and how well it is written (style, form, spelling and grammar).

14. Consider Theorem 3 from the text. Show that if $\beta$ is a multiple of $\alpha$, then the choices of $\rho$ and $\gamma$ are unique.

15. Consider Theorem 3 from the text. Show that if $\rho$ and $\gamma$ are unique, then $\beta$ is a multiple of $\alpha$.

16. Let $\alpha = 5 + \sqrt{2}$, $\beta = 6 - 19\sqrt{2} \in \mathbb{Z}[\sqrt{2}]$. Write $\beta = \alpha \gamma + \rho$ for $\gamma, \rho \in \mathbb{Z}[\sqrt{2}]$ with $|N(\rho)| < |N(\alpha)|$. 