42. Find a cyclic subgroup of $S_3$ and determine all of its generators.

43. Is the group of symmetries of the rhombus cyclic?

44. The text describes how the permutation $(12)(34)$ of a tetrahedron can be formed by rotating the tetrahedron by $180^\circ$ across a line (text, p. 100). Draw and label a tetrahedron, and draw and label that line.

45. Why does a transposition change parity? Give an explanation to your group that is appropriate for a high school student.

46. Can every permutation be written as a product of transpositions? If so, prove it. If not, provide a counterexample.

47. Show that the function $f : S_n \rightarrow S_n$ given by $f(\sigma) = (12)\sigma$ is a one-to-one correspondence. That is, show that:
   
   (a) For every $\tau \in S_n$, there is some $\sigma \in S_n$ with $f(\sigma) = \tau$.
   
   (b) For any $\sigma$ and $\tau$ in $S_n$, if $f(\sigma) = f(\tau)$, then $\sigma = \tau$.

48. Write down the equations for each of the 16 symmetries in $D_8$.

49. If the center of an equilateral triangle is at $(0,0)$ and one vertex is at $(1,0)$, find the locations of the other two vertices.

50. Using the coordinate system from the previous problem, write down the equations for the following symmetries in $D_3$:
   
   (a) The flip around the $x$-axis.
   
   (b) Rotation left by $120^\circ$.
   
   (c) One of the other two flips.