Take Home Exam #2
Math 635
Due: November 20, 2002

Name_________________________

1. Solve the initial value problem

\[
\begin{align*}
y' &= -5y + 5t^2 + 2t \\
y(0) &= \frac{1}{3}
\end{align*}
\]

on the interval \(0 \leq t \leq 5\), using
a. the explicit fourth order Runge-Kutta method
b. the Gauss method of order 4
c. the Lobatto method of order 4.

Compare your solution to the exact solution \(y(t) = t^2 + \frac{1}{3}e^{-5t}\).

2. Solve the problem

\[
\begin{align*}
y'(t) &= \frac{y - y^2}{t} \\
y(1) &= 1
\end{align*}
\]

on the interval \(1 \leq t \leq 10\) using a Runge-Kutta-Fehlberg method and a variable step size. In particular, at each step calculate the residual \(R\) which is the difference between the two calculated solutions. If \(R \leq TOL\) accept the approximation, and increment \(t\) by the step size; if not, then disregard the approximation. We then choose the next step size as follows. Set \(\delta = 0.84 \left( \frac{TOL}{R} \right)^{1/4}\). If \(\delta \leq 0.1\), then set \(h = 0.1h\); else if \(\delta \geq 4\) set \(h = 4h\); otherwise set \(h = \delta h\). If \(h > h_{\text{max}}\) set \(h = h_{\text{max}}\), while if \(h < h_{\text{min}}\) then the program terminates unsuccessfully. Choose appropriate values for the parameters of the problem, and compare your result to the exact solution \(y(t) = \frac{t}{1 + \ln t}\).

3. Solve the system

\[
\begin{align*}
y'(t) &= 1 + \frac{y}{t} + \frac{y^2}{t^2} \\
y(1) &= 0
\end{align*}
\]

on the interval \(1 \leq t \leq 4\) using
a. a fourth-order Adams-Bashforth method
b. a fourth-order Adams-Moulton method
c. a fourth-order Predictor-Corrector method.

Compare your results to the exact solution \(y(t) = t \tan \ln t\).