16. Prove that \( f(x) = \sqrt{x} \) is continuous for all \( x > 0 \).

17. Let
\[
f(x) = \begin{cases} 
\exp(-1/x^2) & \text{if } x > 0, \\
0 & \text{if } x \leq 0.
\end{cases}
\]
Prove that \( f(x) \) is differentiable for \( x = 0 \).

18. Show that, if \( f \) is differentiable at \( x \) that
\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x - h)}{2h}
\]
but that the converse is, in general, false.

19. Use induction to prove that
\[
\sum_{k=1}^{n} k^2 = \frac{k(k+1)(2k+1)}{6}.
\]

20. Let \( f(x) = x^2 \). For the interval \([0, a]\), find \( L(f, n) \) and \( U(f, n) \). Use the definition to prove that \( f \) is integrable on \([0, a]\), and to find the value \( \int_{0}^{a} f(x) \, dx \).

21. Let \( \phi(x) \) be the Dirichlet function
\[
\phi(x) = \begin{cases} 
1 & \text{if } x \text{ is rational}, \\
0 & \text{if } x \text{ is irrational}.
\end{cases}
\]
Prove that the integral \( \int_{0}^{1} \phi(x) \, dx \) does not exist, using Definition 2.4.2.

Comment: Definition 2.4.2 is for the Riemann integral. Other definitions of the integral exist, the most important of which is probably the Lebesgue integral. The Lebesgue integral of the Dirichlet function does exist; see p. 241 of the text.

22. Evaluate
\[
\lim_{n \to \infty} \frac{1^\alpha + 2^\alpha + \cdots + n^\alpha}{n^{\alpha+1}}
\]
for \( \alpha \geq 0 \).

Hint: I sure look like the upper sum of an integral. I wonder if the Fundamental Theorem of Calculus might help.