The use of graphing calculators is permitted. All problems are worth the same amount.

1)  
   a) State precisely the theorem that describes when and to what a Fourier series converges. 
   b) In a homework problem we derived the expansion  
      \[ e^{ax} \approx 2 \sinh a \pi \left[ \frac{1}{2a} + \sum_{n=1}^{\infty} \frac{(-1)^n}{a^2 + n^2} (a \cos nx - n \sin nx) \right] \]  
      for \( a \neq 0 \). What is the value of the series  
      \[ \frac{2 \sinh a \pi}{\pi} \left[ \frac{1}{2a} + \sum_{n=1}^{\infty} \frac{(-1)^n}{a^2 + n^2} (a \cos nx - n \sin nx) \right] \]  
      when \( x = 0 \)? When \( x = \pi/2 \)? When \( x = \pi \)?

2)  
   a) One edge of a square plate is kept at a uniform temperature \( u_0 \), while the other three edges are kept a temperature zero. Without solving a boundary value problem, but by superposition of solutions to like problems to obtain the trivial case in which all four edges are at temperature \( u_0 \), show that the temperature at the center of the square is \( u_0 / 4 \).  
   b) In a homework problem, we showed that the temperature in a solid sphere of radius \( a \), initially at temperature \( f(r) \), whose surface is kept at temperature zero is  
      \[ u(r,t) = \frac{2}{a r} \sum_{n=1}^{\infty} \exp \left\{ -\frac{n^2 \pi^2 k}{a^2} t \right\} \sin \left( \frac{n \pi r}{a} \right) \int_0^a s f(s) \sin \left( \frac{n \pi s}{a} \right) ds . \]  
      If the sphere is 40cm in diameter, the initial temperature is 100°C, and \( k = 0.15 \) in cgs (centimeters-grams-seconds) units, estimate the temperature at the center of the sphere 15 minutes after cooling begins. Explain the degree of accuracy of your estimate. You may use the fact that \( \lim_{\theta \to 0} \sin \theta = 1 \) without comment.

3) Obtain a formal solution to the boundary value problem  
   \[ \begin{cases}  
      u_t(x,t) = k u_{xx}(x,t), \\
      u|_{x=0} = u|_{x=c} = 0, \\
      u|_{t=0} = f(x)  
   \end{cases} \]  
   on the domain \( 0 < x < c \) for all \( t > 0 \).
4) Find the steady temperature of a semicircular plate \(0 \leq \rho \leq a, \ 0 \leq \phi \leq \pi\), where the temperatures on the boundary are as described in the figure below.