1) What is the Fourier series on \((-\pi, \pi)\) for the function \(f(x) = x + x^2\) ?

2) What is d’Alembert’s solution to the initial value problem

\[
y''(x,t) = a^2 y_{xx}(x,t) \\
y(x,0) = f(x) \\
y_t(x,0) = 0
\]

for \(-\infty < x < \infty\) and \(t > 0\)? Suppose \(f(x)\) is given by the graph below and \(a = 1\); sketch a graph of the solution at time \(t = 2\) and time \(t = 4\). Include a scale for your graphs.

3) Let \(\{\phi_n(x)\}_{n=1}^\infty\) be a sequence of functions in the space \(C_p(a,b)\).
   a) What does it mean to say that this sequence is orthonormal?
   b) Give an example of an orthonormal set of functions with an infinite number of members. Show that they are orthonormal.
   c) Given a function \(f \in C_p(a,b)\), find constants \(\gamma_n\) so that the error \(\left\| f - \sum_{n=1}^{N} \gamma_n \phi_n \right\|\) is as small as possible.

4) Find the Fourier sine series and the Fourier cosine series for the function \(f(x) = \pi - x\) on the interval \((0, \pi)\). Assuming that the both series converge to the function on \((0, \pi)\), sketch graphs of the Fourier sine series and Fourier cosine series on the interval \((-2\pi, 2\pi)\).