Write a C++ program that simulates the spread of HIV through the body. Include the effect of a RT inhibitor.

We shall suppose initially that the patient has virus in the body, but that no T cells have yet been infected.

Representative values for the parameters are given below. Units are microliters (μL) and days.

- $T$ - the number of T cells. Initial value is $T_0 = 1000 \, \mu\text{L}.$
- $T^* -$ the number of infected T cells. Initial value is $T^* = 0 \, \mu\text{L}.$
- $V$ - the number of virus particles. Initial value is $V_0 = 10^{-3} \, \mu\text{L}.$
- $s$ - the rate at which new T cells are generated in the body. $s = 10 \, \text{day}^{-1} \mu\text{L}.$
- $T_{\text{max}}$ - the maximum number of T cells in the body. $T_{\text{max}} = 1500 \, \mu\text{L}.$
- $d_r$ - the natural death rate of uninfected T cells. $d_r = 0.02 \, \text{day}^{-1}.$
- $\delta$ - the death rate of infected T cells. $\delta = 0.24 \, \text{day}^{-1}.$
- $c$ - the clearance rate of the virus. $c = 2.4 \, \text{day}^{-1}.$
- $k$ - the infection rate. $k = 2.4 \times 10^{-3} \, \text{day}^{-1}.$
- $N$ - the number of virus particles produced during the lifetime of an infected T cell. $N = 1200.$
- $p$ - the rate of proliferation of T cells. $p = 0.03.$

1. What is the short-term behavior of the system?
2. Use your simulation to show that $T, T^*, \text{ and } V$ tend to constants as $t \to \infty$; say $T \to T_\infty, T^* \to T^*_\infty, \text{ and } V \to V_\infty.$ Estimate these values.

3. Explain analytically why $T_\infty = \frac{c}{Nk}, V_\infty = \frac{N}{c} \frac{V_0}{N} + \frac{p}{k} - \frac{pc}{Nk^2 T_{\text{max}}}, \text{ and } T^*_\infty = \frac{c}{N\delta} V_\infty.$

4. Does your simulation bear this out?

Now let us analyze the effect of an RT inhibitor. Modify the preceding values as follows. Set

- $T = 83.33 \, \mu\text{L}.$
- $V_0 = 5347 \, \mu\text{L}.$
- $T^* = 44.56 \, \mu\text{L}.$

Answer the following.

5. Use your simulation to show that there is a critical value of the effectiveness parameter $\eta$, say $\eta_{\text{crit}}$, so that if $\eta > \eta_{\text{crit}}$, then $T^* \to 0$ and $V \to 0$ as $t \to \infty.$ Estimate this value.
6. Show that if $\eta < \eta_{\text{crit}}$, then that $T$, $T^*$, and $V$ tend to constants as $t \to \infty$; say $T \to T_{x,\eta}$, $T^* \to T^*_{x,\eta}$, and $V \to V_{x,\eta}$.

7. Doctors say AIDS sets in if $T < 200$. Show that there is a critical value $\eta_{AIDS}$ so that if $\eta > \eta_{AIDS}$, then $T_{x,\eta}^* \geq 200$. Use your simulation to estimate this value.

8. Find analytically the value of $\eta_{AIDS}$. Compare it to the results of your simulation.

You are then to write up a technical report that answers these questions. The report should describe the model, the numerical methods used to solve the problem, your program, and your results. When answering these questions, you must address the question of how the choice of step size affects the result.

The program should be written using good object oriented programming techniques.

Your grade for the project will be based on the following criteria:

- The quality of your program
- The quality of your results
- The quality of your written report