Name____________________________

Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam.

1. State precisely the incidence axioms.

2. State precisely the betweeness axioms.

3. State precisely the congruence axioms.

4. Define the following terms, or indicate that they are undefined.
   a. point
d. ray
   b. line
e. angle
   c. segment f. between
   g. opposite sides of a line

5. Give a model of incidence geometry with a finite number of points that satisfies the elliptic parallel property. Give a second model of incidence geometry with a finite number of points that satisfies the hyperbolic parallel property. Give a third model of incidence geometry that satisfies the Euclidean parallel postulate.

6. Prove that, in incidence geometry, that there exist three lines that are not concurrent.

7. State and prove the line separation property.

8. State and prove the crossbar theorem.

9. A set $S$ is convex if, whenever two points $A$ and $B$ are in $S$, then the segment $AB$ is in $S$. Show that the interior of an angle is convex.

10. Let $S^2$ be the set of all points $(x, y, z)$ in $\mathbb{R}^3$ so that $x^2 + y^2 + z^2 = 1$. Define the relationship $\sim$ on $S^2$ by $(x, y, z) \sim (-x, -y, -z)$. Show that $\sim$ is an equivalence relation. Let the points of $P^2$ be the set of equivalence classes of $S^2$, let lines in $P^2$ be the image of great circles under $\sim$, and let incidence be defined naturally. Show that $P^2$ is a projective plane.