Name______________________________

Do all of your work on the blank paper that has been provided.

§1 Calculation

1. Find $\nabla f$ and the second partials $f_{xx}$ and $f_{xy}$ for $f(x, y) = 3xy^2 - 2y + 5x^2y^2$.

2. Find an equation of the tangent plane to the hyperboloid $z^2 - 2x^2 - 2y^2 = 12$ at the point $(1,-1,4)$.

3. Find the local maximum and minimum values and saddle points of $f(x, y) = x^4 + y^4 - 4xy + 1$.

4. Use Lagrange Multipliers to find the maximum value of $f(x, y) = 4xy$ subject to the constraint $\frac{x^2}{9} + \frac{y^2}{16} = 1$.

§2 Comprehension

5. What is the gradient of a function? What is the directional derivative of a function? How can you use the gradient to calculate the directional derivative? Illustrate your ideas with an example.

6. Give an informal and a precise definition of the limit $\lim_{(x,y)\to(a,b)} f(x, y)$. If $\lim_{x\to0} f(x,0) = L$, and $\lim_{y\to0} f(0,y) = L$ can we conclude that $\lim_{(x,y)\to(0,0)} f(x,y) = L$? Explain your answer.

§3 Application

7. The surfaces $x^2 + y^2 + z^2 = 6$ and $x - y - z = 0$ intersect at $(2,1,1)$. Find the symmetric equations of the tangent line to the curve of intersection of the two surfaces at given point. Find the angle between the gradient vectors at this point, and explain its geometric significance.

8. Suppose that $w = f(x, y)$ where $x = r \cos \theta$ and $y = r \sin \theta$. Write $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial \theta}$ in terms of $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$. Use this to simplify the expression $\left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial w}{\partial \theta}\right)^2$.

9. Show that the rectangular box of maximum volume inscribed in a sphere or radius $r$ is a cube.
The equation $3x^2 z - x^2 y^2 + 2z^3 + 3yz = 5$ defines $z = z(x, y)$ implicitly as a function of $x$ and $y$; the result is graphed below. Starting at the point $x = 0$, $y = 1$, in which direction would you travel so that $z(x, y)$ increases as rapidly as possible? How fast would $z$ increase in that direction? Note that if $x = 0$, and $y = 1$, then $z = 1$. 