Midterm Examination #3
Math 273 Calculus 1
Wednesday, April 18, 2001

Name___________________________

§1 Computation:

1) Differentiate. Simplify your answer.
   a) \( f(x) = 4\pi^2 \)
   b) \( f(x) = e^{x^3} + 1 \)
   c) \( f(x) = x\sqrt{x} + \frac{1}{x^2\sqrt{x}} \).

2) Differentiate. Simplify your answer
   a) \( y = \cos(a^3 + x^3) \)
   b) \( y = \left( \frac{x - 6}{x + 7} \right)^3 \)
   c) \( y = \tan(x\sin x) \).

3) Find the equation of the tangent line to the astroid \( x^{2/3} + y^{2/3} = 4 \) that passes through the point \( (-3\sqrt{3}, 1) \).

4) Differentiate \( y = x^{\sqrt{x}} \). Simplify your answer.

§2 Comprehension:

5) What is the sum rule for differentiation? Prove it.

6) Use implicit differentiation to find the derivative for \( \sin^{-1} x \).

7) Here is the graph of a function, its first derivative, and its second derivative. Identify each.
§3 Applications:

8) Newton’s Law of Gravitation says that the magnitude $F$ of the force exerted by a body of mass $m$ on a body of mass $M$ is $F = \frac{GmM}{r^2}$ where $G$ is the gravitational constant and $r$ is the distance between the bodies.

a) If the bodies are moving, find $\frac{dF}{dr}$, and explain its meaning. What does the sign of $\frac{dF}{dr}$ indicate?

b) Suppose that it is known that Earth attracts an object with a force that decreases at the rate of 2 N/km when $r = 20,000$ km. How fast does this force change when $r = 10,000$ km?

9) A plane flying horizontally at an altitude of 1 mi and a speed of 600 mi/hr passes directly over a radar station. Find the exact rate at which the distance from the plane to the radar station is increasing when it is 3 mi away from the station.

10) A man starts walking north at 5 ft/s from a point $P$. Five minutes later, a woman starts walking south at 4 ft/s from a point 500 feet due east of $P$. At what rate are the people moving apart 15 min after the woman starts walking?