Exam #3  
Math 119  
April 23, 2004

Name______________________________

All questions are worth an equal number of points. All work is to be done on the blank paper provided. At the end of the exam, please hand in this sheet, together with all of your work. The use of calculators is prohibited.

§1 Calculation

1. Evaluate each of the following.
   a. \( \tan^{-1} \sqrt{3} \)
   b. \( \sin \frac{\pi}{4} \)
   c. \( \cos \frac{5\pi}{6} \)
   d. \( \cos^{-1} 0 \)

2. Simplify each of the following
   a. \( \ln e^{2x+1} \)
   b. \( 10^{\log_{10}(x-5)} \)
   c. \( \log_{49} 7 \)
   d. \( \ln \sqrt{\frac{e^{x^2}}{2}} \)

3. Solve \( (\ln x)^2 = \ln x^2 \).

4. Sketch a graph of the following functions. [You may include them all on the same graph if you wish.]
   a. \( y = \sin x \)
   b. \( y = \cos x \)
   c. \( y = \tan x \)
   d. \( y = \sec x \)

5. Evaluate exactly \( \cos(\sin^{-1} x) \).
6. A right triangle is shown below. Find all the angles and all the lengths.

\[ \begin{align*}
&\text{a=}? \\
&\text{h=}? \\
&\text{s=}? \\
\end{align*} \]

\begin{align*}
&\text{12} \\
&40^\circ
\end{align*}

\[ \text{\textdegree} \]

\section*{2 Comprehension}

7. What is the definition of \( \sin \theta \) and \( \cos \theta \)? Use your definition to prove that \( \sin^2 \theta + \cos^2 \theta = 1 \) for all \( \theta \).

8. What is the definition of \( \sin^{-1} x \)? Is it always the case that \( \sin \sin^{-1} x = x \)? Is it always the case that \( \sin^{-1} \sin x = x \)? Explain.

\section*{3 Application}

9. The Richter scale says that the magnitude \( M \) of an earthquake is \( M = \frac{2}{3} \log_{10} \frac{E}{E_0} \) where \( E \) is the amount of energy released, and \( E_0 = 10^{4.40} \text{J} \). The 1933 Long Beach, California, earthquake had a Richter scale reading of 6.3, while the 1964 Anchorage, Alaska, earthquake had a Richter scale reading of 8.3. How many times more powerful was the Anchorage earthquake than the Long Beach earthquake?

10. A belt connects a pulley of 2 inch radius with a pulley of 5 inch radius. If the larger pulley turns through 10 radians, how far does the smaller pulley turn?