

MATH 675

Asymptotic and Perturbation Analysis

Class Policies

Mike O'Leary

Office: YR 317

Office Phone: 410-704-4757

Email: moleary@towson.edu

Office Hours: TR 10-11

Fall 2021

Class: Tu 5:00 - 7:40

Room: YR 216

Section: 101

Prerequisites: Math 374/574 or equivalent and Math 475/577 or equivalent

Catalog Description: Asymptotic series and asymptotic methods for approximating solutions to linear and nonlinear ordinary differential equations. Asymptotic expansion of integrals; Watson's Lemma. Perturbation series; regular and singular perturbation theory. Boundary layer theory for ordinary differential equations.

Instructional Material The primary text is

- C. Bender & M Orszag, *Advanced Mathematical Methods for Scientists and Engineers*, Springer-Verlag, 1999.

Other useful references include:

- M. van Dyke, *Perturbation Methods in Fluid Mechanics*, The Parabolic Press, 1975.
- E.J. Hinch, *Perturbation Methods*, Cambridge University Press, 1991.
- M.H. Holmes, *Introduction to Perturbation Methods*, Springer-Verlag, 1995.
- S. Howison, *Practical Applied Mathematics*, Cambridge University Press, 2005.
- P.D. Miller, *Applied Asymptotic Analysis*, American Mathematical Society, 2006.
- A.H. Nayfeh, *Perturbation Methods*, John Wiley & Sons, 2000.
- F.W.J. Olver, *Asymptotics and Special Functions*, A K Peters, 1997.
- J.G. Simmonds & J.E. Mann Jr., *A First Look at Perturbation Theory*, Dover Publications, 1998.

Course Topics: The following is the planned list of course topics (with the corresponding section in our primary text); we may add or subtract material depending on the balance of available time in class.

1. Ordinary Differential Equations (Chapter 1, Sections 1-7). Ordinary differential equations, initial-value and boundary-value problems, theory of homogeneous linear equations, solutions of homogeneous linear equations, inhomogeneous linear equations, first-order nonlinear differential equations, higher-order nonlinear differential equations, eigenvalue problems, differential equations in the complex plane.
2. Approximate Solution on Linear Differential Equations (Chapter 3, Sections 1-8). Classification of singular points of homogeneous linear equations, local behavior near ordinary points of homogeneous linear equations, local series expansions about regular singular points of homogeneous linear equations, local behavior at irregular singular points of homogeneous linear equations, irregular singular points at infinity, local analysis of inhomogeneous linear equations, asymptotic relations, asymptotic series

3. Asymptotic Expansion of Integrals (Chapter 6, Sections 1-5). Introduction, elementary examples, integration by parts, Laplace's method, Watson's Lemma, method of stationary phase.
4. Perturbation Series (Chapter 7, Sections 1-4). Perturbation theory, regular and singular perturbation theory, perturbation methods for linear eigenvalue problems, asymptotic matching.
5. Summation of Series. (Chapter 8, Sections 1-3). Improving convergence; Richardson extrapolation. Summation of divergent series via Euler, Borel, and generalized Borel summation. Padé summation.
6. Boundary Layer Theory (Chapter 9, Sections 1-6). Introduction to boundary layer theory, mathematical structure of boundary layers: inner, outer, and intermediate limits, higher order boundary layer theory, distinguished limits and boundary layers of thickness $\neq \epsilon$.
7. (Time permitting) WKB Theory (Chapter 10, Sections 1-3). The exponential approximation for dissipative and dispersive phenomena, conditions for the validity of the WKB approximation, patched asymptotic approximations, WKB solutions inhomogeneous linear equations.

Attendance: Attendance is expected; you should only miss a class for a compelling reason. If you do miss a class, you are responsible for any material that you miss, including any homework assignments given in that class. Unexcused absences can result in a lower grade.

Students should not attend classes or other university events from the onset of flu-like symptoms until at least 24 hours after the fever subsides without the use of fever reducing medications. Such absences will be considered excused absences; however, students are responsible for the material covered during the period of their absence.

Grading: Problem sets will be assigned each week; they will be collected 1-2 weeks later. There will be no midterm examinations, but there will be a final exam. The final grade shall be determined by a weighted average of the homework grade and the final exam grade; the final exam is worth 30% and the homework assignments will be worth 70%.

Guidelines for Homework:

1. Late work will not be accepted without a compelling reason.
2. Assignments are required to be neat, clean, and paper-clipped or stapled.
3. Assignments must include the author's name, and a brief description of the assignment.
4. Students are allowed to discuss homework problems with their classmates, however all work that is turned in must be the student's own work.

Any assignment that does not meet these criteria may receive a deduction in score, or more generally will simply be rejected.

Final Exam: The Final Exam is scheduled for Tuesday, December 21, from 5:00 - 7:00 pm; please note the time. The final exam will not be rescheduled. Attendance is expected; a make-up exam will not be given without an extremely compelling reason. The final exam shall be comprehensive.

Academic Integrity The nature of higher mathematics requires that students adhere to accepted standards of academic integrity. Violations of academic integrity include cheating, plagiarism, falsification and fabrication, complicity in academic dishonesty, personal misrepresentation and proxy, bribes, favors and threats. Cheating is a serious offense that will have grave consequences for your academic life.

Students who violate these standards will either fail the course outright or, at the instructor's discretion, may merely receive a zero on any assignment for which the student receives inappropriate assistance. Particularly serious violations of these standards will be referred to the administration for possible additional action.

Withdraw: The last day to withdraw from the course with a grade of 'W' is November 8.

Masks: The University has set clear expectations about mask wearing for the safety of all in our community. Face coverings over the nose and mouth are required at all times while indoors. There are no exceptions to this rule. Students not wearing a mask—or wearing a mask improperly—will be asked to wear a mask or to fix their mask position. Any student refusing to put on or wear a mask will be asked to leave the classroom immediately. Students who have additional incidents with the mask expectation after a first warning will be referred to the Office of Student Conduct for failure to comply with this University directive.

Help: If you have difficulty completing a homework assignment, do not hesitate to ask for help, either from your friends, or from me. You are welcome to stop by my office, for whatever reason, and at whatever time, even if there are no office hours scheduled then. If you wish, you may also simply send an e-mail message.