WHERE IS THE FIFTH DIMENSION?

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1 June 2015

Abstract: Recent advances show that a fifth dimension leads to substantial improvement in our understanding of cosmology and particle physics, so instead of continuing to argue about why we do not "see" it, I suggest that the fifth dimension be regarded as a *concept* on par with that of time.

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The existence or otherwise of extra dimensions – and the fifth dimension in particular – is a vintage problem in physics. This problem has recently become acute: new results show that even one extra dimension leads to a substantial improvement in our theoretical understanding of cosmology and particle physics, but apparently we still cannot "see" this elusive fifth dimension. My purpose here is to examine recent advances in the theory of the fifth dimension, and to suggest a new way of regarding it.

It is well known that in 1921 Kaluza showed it was possible to unify gravity with classical electromagnetism by adding a fifth dimension to Einstein's theory of general relativity [1]. To account for the fact that physics appeared to operate in a *four*-dimensional world of space and time, Kaluza assumed that quantities such as potentials did not depend on the extra coordinate (the "cylinder condition"). The theory was extended to quantum phenomena in 1926 by Klein, who showed it was possible to explain the unique value of the charge on the electron by assuming that, while the particle was free to move in space and time, it was *trapped* in the extra dimension because the latter was closed on itself [2]. To account for the non-observability of the fifth dimension, Klein assumed that it was rolled up to an infinitesimal size ("compactification"). In subsequent years, 5D Kaluza-Klein theory evolved through several stages. Then in the 1990s, it was realized by two separate groups of workers that a truly effective theory of higher-dimensional relativity should not be hobbled by the cylinder condition and compactification, but should enjoy the physical flexibility which follows from unconstrained 5D algebra.

The two modern versions of 5D relativity are known as Space-Time-Matter theory [3, 4] and Membrane theory [5, 6]. Much has been written on both but a review of recent results is available [7]. In STM theory, the extra dimension and derivatives of the potentials with respect to the extra coordinate form a quantity which is identical in its properties to the energy-momentum tensor of 4D general relativity. This result actually follows from an old embedding theorem due to Campbell, about which more will be learned below. It means that matter has a geometrical origin. In M theory, the membrane is a singular hypersurface in five dimensions, which effectively concentrate the interactions of particles, accounting for why they are stronger than gravity (which can propagate freely outside the membrane and into the "bulk"). The structure represented by the membrane also helps to understand why the masses of elementary particles are much less than the Planck mass, which is of order 10^{-5}g and might otherwise be the preferred mass in a theory based on Planck's constant and the gravitational constant. It should be noted that while STM theory and M theory have different physical notations, they are mathematically similar. In what follows, I will adopt the STM approach, but the two theories are in many respects complementary.

Most accounts of 5D relativity employ field equations based on the 5D Ricci tensor. This contains the conventional 4D Ricci tensor, so the 5D field equations may be cast in a form resembling those of general relativity but modified by the presence of extra terms. The same argument applies to the 5D equations of motion for a test particle, which resemble the 4D geodesics of Einstein's 4D theory with extra terms (see below). Thus the 5D theory smoothly embeds the standard 4D theory. An important consequence of this is that the 5D theory agrees in the approximate limit with observational tests of 4D general relativity.

There are, of course, differences; and some striking ones occur in cosmology. The 5D analogy of the standard 4D models of cosmology were discovered by Ponce de Leon in 1988, including the 5D versions of the Einstein-de Sitter (dust) model and the de Sitter (vacuum) model. The latter had been shown by pioneering work of Robertson in 1928 to be equivalent via coordinate transformations to 5D Minkowski space. That is, the de Sitter model was realized to be curved in 4D but flat in 5D. This appears to have been regarded as a mathematical quirk with no practical implications at the time, since the de Sitter model contains only vacuum energy and no ordinary matter. However, work by members of the STM group showed in 2001 that the 5D matter-filled Einstein-de Sitter model could also be changed into the 5D Minkowski form, albeit via a complicated coordinate transformation. Later work by several authors went on to show that all of the standard Friedmann-Robertson-Walker models are in fact flat in 5D even though they are curved in 4D. This intriguing result can be appreciated in an elementary manner by taking a sheet of paper and rolling it into a cone, where the point represents a singularity even though the sheet is intrinsically flat. In more advanced terminology, we see that the singularity at the time origin of the FRW models is due to a poor choice of 4D coordinates in a flat 5D manifold. In other words, if the universe is 5D in nature then it is inherently flat, and the big bang is a 4D artefact.

This inference is no doubt satisfying to those researchers who have traditionally been suspicious of the singularities in Einstein gravity. Another aspect of general relativity which has been the focus of controversy concerns the cosmological constant. In Einstein's theory, this measures the energy density of the vacuum, and is a true constant. Observations indicate that the magnitude of the cosmological constant is small as measured over large scales. But models of elementary particles indicate that their stability requires the presence of vacuum fields, which correspond in a formal sense to values of the cosmological constant which are large on small scales. This conflict which is now well-known as the cosmological-constant problem, finds no reasonable resolution in 4D. However, it can in principle be resolved in 5D. The reason, as will be seen below, is that in STM theory the size of the cosmological 'constant' varies in general, being dependent on the local values of the extra coordinate. It should also be mentioned that 5D cosmological models have been found in which the cosmological 'constant' decays with time. In these models, the energy density of the vacuum is large at early times, as required by quantum-field calculations of inflation; and the density falls as the universe expands, to levels in agreement with observations made at the present epoch. Supernova data at high redshifts may, with a modest improvement in accuracy, soon be able to differentiate between 4D and 5D models of the universe.

Present knowledge indicates that vacuum energy is important not only on cosmological scales but also on particle scales. For example, the small radiative shifts in the energy levels of atoms first observed by Lamb in 1947 is nowadays interpreted in terms of energetic processes in the vacuum. What does the 5D approach have to say about the nature of the vacuum on small scales?

In Space-Time-Matter theory, there is an exact solution of the 5D field equations which, because of its basic nature, is known as the canonical metric. It is a corollary of Campbell's theorem that any solution of Einstein's 4D field equations, empty of ordinary matter but with vacuum energy measured by a cosmological constant, can be embedded in the 5D canonical solution. The 5D metric which expresses the solution, involves the following quantities: the 5D interval is dS^2; which includes the conventional 4D one ds^2; the extra coordinate can be labelled l, which may in general be subject to a constant shift $l \to (l - l_0)$; the scale of the potential associated with the cosmological constant Λ is measured by the constant length L; and the spacelike or timelike nature of the extra dimension is indicated by $\varepsilon = \pm 1$, both being allowed. Then the (shifted) canonical metric is

$$dS^2 = \frac{(l - l_0)}{L^2} ds^2 + \varepsilon dl^2 \;, \qquad \Lambda = -\frac{3\varepsilon}{L^2}\left(\frac{l}{l - l_0}\right)^2 \quad . \tag{1}$$

This metric has a considerable literature [7]. Much of it concerns the 5D null paths, defined by (1) with $dS^2 = 0$. Indeed, the 5D condition $dS^2 = 0$ is frequently taken in place of the 4D one $ds^2 \geq 0$ as defining causality in the higher-dimensional manifold. This is not the place to go into details, but there are two major consequences of (1) which may be verified either directly from that relation or by referring to the original works. They both involve small-scale physics, and may be summarized as follows:

(a) The path $l = l(s)$ along the extra coordinate as a function of 4D proper time is monotonic if the extra dimension is spacelike ($\varepsilon = -1$) or oscillating if the extra dimension is timelike ($\varepsilon = +1$; see Fig. 1). The 4D curvature, as measured by the cosmological constant, is positive ($\Lambda > 0$) or negative ($\Lambda < 0$) respectively. Assuming that both modes occur together, there results a simple description of wave-particle duality in a nearly-flat spacetime.

(b) There is a divergence in the energy density of the vacuum as measured by $|\Lambda|$ at $l \to l_0$ which is qualitatively similar to the membrane of M theory (see Fig. 2). However, while the divergent hypersurface in (1) may as well be called a membrane, it is distinct in several ways: it is not truly singular because the waves in $l(s)$ can pass through $l=l_0$; and the hypersurface at l_0 arises naturally from the physics of the metric (1), rather than being inserted "by hand" as in M theory. In fact, the physics around $l=l_0$ can be investigated in detail, with some interesting implications [7]. Prime among these is that the waves of the timelike $(\varepsilon=+1)$ node have properties similar to those of de Broglie waves. This suggests that the constant *L* in (1) be identified as the Compton wavelength of the particles associated with the wave so $L=h/mc$. (Here *h* is Planck's constant, *m* is the particle mass and *c* is the speed of light.) With this assumption, it may be shown that near the membrane ($l\rightarrow l_0$) the action becomes quantized with $mds\approx h$.

The preceding two consequences of the canonical metric (1) are remarkable. And there are other interesting topics, such as the quantum uncertainty for interacting particles and the properties of the matter associated with more complicated metrics, which invite study. However while the addition of an extra dimension is profitable in that it gives a significant improvement in our understanding, we are brought inevitably to the vintage question: Why do we not *see* the fifth dimension?

There *is* a reasonable answer to this question, despite its age. But it requires us to clarify our thinking processes as they apply to physics. Specifically, our subject deals with both objects and concepts. The former we can in some sense "see"; while the latter we cannot, though we are usually able to cogitate about them. As a simple example, consider *time*. The symbol for this appears in most of the equations of physics, but we cannot actually "see" time. There are also more complicated examples that have challenged great thinkers through the ages. Newton could not "see" gravity when he formulated the inverse-square law for it. And Einstein could not let his eye rest on the curvature of spacetime when he used the concept to reformulate the force of gravity. The astronomer Eddington, who was a contemporary of Einstein, was the first scientist of stature to point out that much of what physics deals with is conceptual in nature, and not concrete enough to see.

As regards the fifth dimension, we can infer from its consequences that it has to do with an origin of matter and the masses of particles (though to fully understand these will require better knowledge of the scalar field generally associated with the extra coordinate). But we should no more expect to "see" the fifth dimension than we can time. It is possible to accept the fifth dimension, not as a theory but as a useful concept.

Acknowledgements

I am grateful for comments to several members of the Space-Time-Matter group (5Dstm.org).

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Figures:

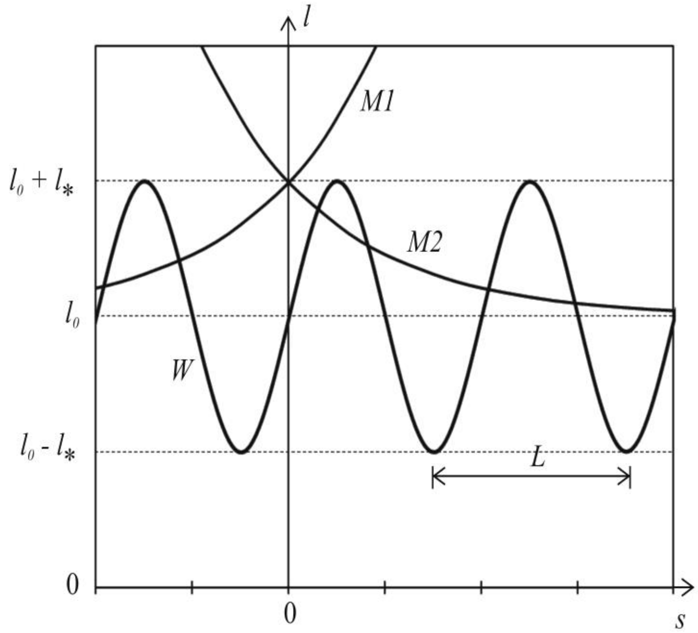

Fig. 1. The behaviour of the extra coordinate *l* as a function of the 4D proper time *s*, for the null 5D geodesics of the (shifted) canonical metric (1) defined by $dS^2 = 0$. The paths are monotonic (M1, M2) or wavelike (W), depending on whether the extra dimension is spacelike ($\varepsilon = -1$) or timelike ($\varepsilon = +1$). There are two types of monotonic paths because the motion in the extra dimension is reversible. The wave has locus l_0, amplitude l_* and wavelength L. [W is given by $l = l_0 + l_* \exp(\mathrm{i}\, s/L)$, M1 by $l = l_0 + l_* \exp(s/L)$ and M2 by $l = l_0 + l_* \exp(-s/L)$.]

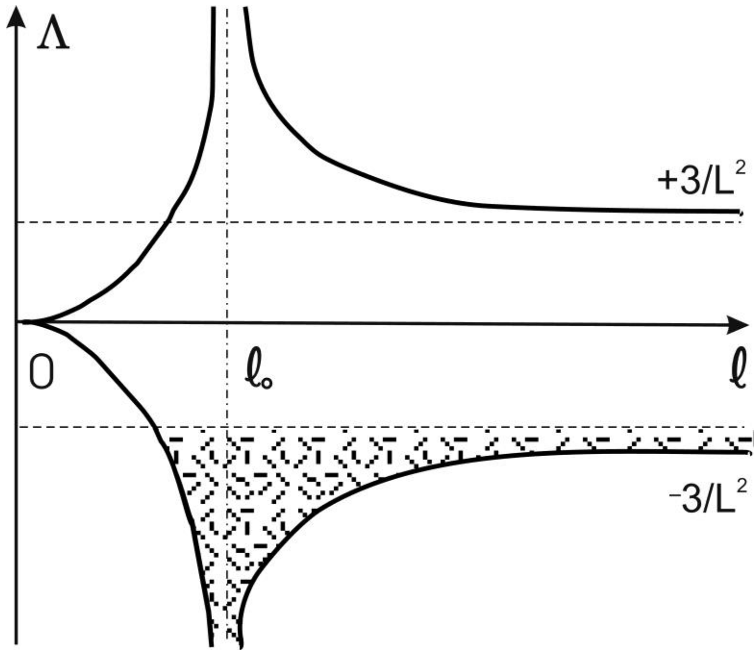

Fig. 2. A schematic plot of the cosmological 'constant' Λ as a function of the extra coordinate l, according to Equation (1). Λ can be positive or negative depending on whether the extra dimension is spacelike ($\varepsilon = -1$) or timelike ($\varepsilon = +1$). It diverges for $l \to l_0$ and is asymptotic to $\pm 3/L^2$, where l_0 is the shift in the canonical metric and L determines the scale of the spacetime part. The stippled region is a trap for the wave illustrated in Fig. 1.