

Constraints on Kaluza-Klein Gravity from Gravity Probe B

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Received: date / Accepted: date

Abstract Using measurements of geodetic precession from Gravity Probe B, we constrain possible departures from Einstein's General Relativity for a spinning test body in Kaluza-Klein gravity with one additional space dimension. We consider the two known static and spherically symmetric solutions of the 5D field equations (the soliton and canonical metrics) and obtain new limits on the free parameters associated with each. The theory is consistent with observation but must be "close to 4D" in both cases.

Keywords higher-dimensional gravity · experimental tests of gravitational theories

PACS 04.50.-h · 04.80.Cc

1 Introduction

There is now a substantial literature on the higher-dimensional extension of Einstein's general theory of relativity known as Kaluza-Klein gravity [1,2]. There are several ways to test the theory, with perhaps the most straightforward involving the motion of test objects in the field of a static, spherically-symmetric mass like the Sun or the Earth. Birkhoff's theorem in the usual sense does not hold in higher dimensions [3,4,5], so some question arises about the best choice of metric to describe such a situation. But the gravitational field

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of the Sun must be close to Schwarzschild, by the solar system tests. Also, the non-linearity of Einstein's equations requires us to use exact solutions to model the field. In the minimal five-dimensional (5D) case, only two such are known: the soliton [6, 7, 8] and canonical solutions [9, 10, 11]. Both satisfy the vacuum field equations in 5D, consistent with the spirit of Kaluza's original idea that 4D matter and gauge fields appear as a manifestation of pure geometry in the higher-dimensional world. The soliton metric contains no explicit ℓ -dependence and reduces to the standard 4D Schwarzschild solution on hypersurfaces $\ell = \text{const}$. The canonical metric contains explicit ℓ -dependence; but its effects are suppressed by a quadratic prefactor $(\ell/L)^2$ where L is a constant and presumably a large length scale (a free parameter of the theory). The fifth dimension in this solution is flat.

The classical tests of general relativity were first applied to the soliton by Kalligas et al. and Lim et al. in 1995 [13, 14], and again in more generality by Liu and Overduin in 2000 [15]. Light deflection, perihelion precession of Mercury, and radar ranging to Mars set limits of order 10^{-2} on the primary free parameter of the metric, and the hope was expressed that data on geodetic precession for spinning test masses from Gravity Probe B might push this down to as little as 10^{-4} . Overduin then used observational constraints on violations of the equivalence principle to obtain bounds of order $10^{-6} - 10^{-8}$ on the soliton metric as applied to the Sun, Earth, Moon and Jupiter [16]. If the soliton metric were the only choice available, such a result might call the need for a higher-dimensional extension of general relativity into question, contrary to what is suggested by most attempts to unify gravitation with the other fundamental interactions. However, analysis of the canonical metric by Mashhoon, Wesson and Liu [11] has revealed that for this solution, the classical tests are satisfied *exactly* for non-spinning test bodies. This is due to the flatness of the extra dimension and the quadratic prefactor on the 4D part of the metric and can be proven using a 1926 theorem on embedding by Campbell; see Ref. [12] for discussion.

Spin thus emerges as a potentially critical discriminator between standard and higher-dimensional extensions of general relativity, and the canonical solution may be the most appropriate for this problem. The geodetic effect for the canonical metric was first worked out by Liu and Wesson in 1996 [10]. We return to this work and assess the status of the theory using the recently released final results from Gravity Probe B (henceforth GPB [18]). The geodetic effect is briefly reviewed in Section 2. We consider the soliton metric in Section 3 and the canonical metric in Section 4. Our results are summarized and discussed in Section 5.

2 Geodetic effect

The geodetic effect is the first test of general relativity to involve the spin of the test body (the other being the frame-dragging or Lense-Thirring effect) and was originally investigated by Willem de Sitter in 1916 using the orbital

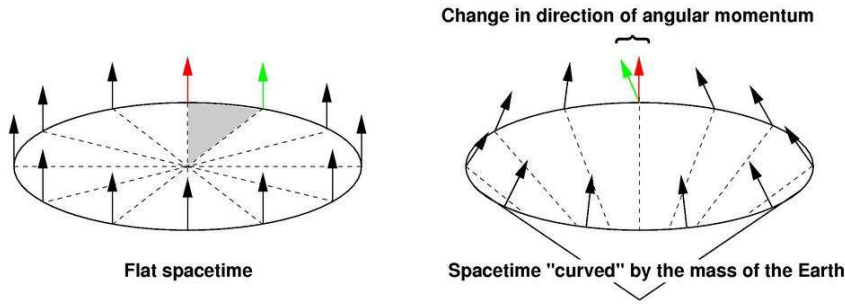


Fig. 1 The “missing inch” model for geodetic precession. A gyroscope’s spin vector (arrow) is orthogonal to the plane of its motion, and in flat spacetime its direction is unchanged as the gyroscope completes an orbit. If, however, space is folded into a cone to simulate curvature due to the earth (right), then part of the area inside the circle (shaded at left) must be removed and the gyroscope’s spin vector no longer lines up with itself after making a complete circuit. This angular shift contributes two-thirds of the total geodetic effect, and the difference between the circumference of the orbit with and without this effect at GPB’s operating altitude of 642 km is about an inch [20].

angular momentum of the Earth-Moon system as a “gyroscope” in the field of the Sun (this is now termed the “solar geodetic” or de Sitter effect and was also observed by GPB [19]). De Sitter’s calculations were extended to include spin angular momentum of rotating test bodies such as the earth by Jan Schouten in 1918 and Adriaan Fokker in 1920. Thorne [20] has shown how the effect may be thought of as arising from two separate contributions: one due to space curvature around the central mass and the other a spin-orbit coupling between the spin of the gyroscope and the “mass current” of the central mass (in the rest frame of the orbiting gyroscope). The space curvature effect is twice as large as the spin-orbit one. It arises because the gyroscope’s angular momentum vector, orthogonal to the plane of the motion, no longer lines up with itself after one complete circuit through curved spacetime around the central mass (Fig. 1). The spin-orbit contribution is analogous to Thomas precession in classical electromagnetism (where the electron experiences an induced magnetic field due to the motion of the nucleus relative to its own rest frame). Both contributions together produce what is known as the geodetic effect within general relativity, causing the gyroscope’s spin axis to precess by a predicted 6606 milliarcseconds (mas) per year in the North-South direction for a polar orbit at GPB’s altitude of 642 km [21, 22, 23].

To treat this problem in extended theories of gravity, one begins with an appropriate choice of metric and solves the equations of motion for the velocity (geodesic equation) and angular momentum (parallel transport equation) of the test body, assuming that the two vectors are orthogonal. The geodetic effect is the excess of the test body’s spin angular velocity over its orbital angular velocity. Our notation follows that in Refs. [10, 15] except that we restore physical units and label the extra coordinate $x^4 = \ell$. Lowercase Greek

indices α, β, \dots run over 0, 1, 2, 3 as usual while uppercase Latin indices A, B, \dots run over all five indices 0 – 4. Proper distance in 5D (dS) is related to its 4D counterpart (ds) by $dS^2 = ds^2 + g_{\ell\ell} d\ell^2$ so that $d/dS = (ds/dS)d/ds = \sqrt{1 - g_{\ell\ell}(d\ell/dS)^2} d/ds$.

3 Soliton Metric

The line element for the soliton reads (following Ref. [7] but switching to non-isotropic form and defining $a \equiv 1/\alpha$, $b \equiv \beta/\alpha$ and $M \equiv 2m$):

$$dS^2 = A^a c^2 dt^2 - A^{-a-b} dr^2 - A^{1-a-b} r^2 (d\theta^2 + \sin^2 \theta \phi^2) - A^b d\ell^2, \quad (1)$$

where $A(r) \equiv 1 - 2GM/c^2 r$, $M = M_g/a$ where M_g is the gravitational or Tolman-Whittaker mass of the soliton at $r = \infty$, and a and b are free parameters related by a consistency relationship $a^2 + ab + b^2 = 1$ that follows from the field equations. We take b as the primary free parameter of the theory in what follows, noting that the 4D Schwarzschild metric is recovered on hypersurfaces $\ell = \text{const}$ in the limit $b \rightarrow 0$ (and $a \rightarrow +1$). The consistency relation imposes an upper limit of $|b| \leq 2/\sqrt{3} \approx 1.15$. The 4D induced matter associated with this solution has a density proportional to $-ab$ [12],¹ so a and b must have opposite signs. Thus we are restricted *a priori* to values of b in the range $-1 \lesssim b \leq 0$.

The motion of a spinning test body with velocity $u^C \equiv dx^C/dS$ and angular momentum S^C is governed by three central equations; namely, the geodesic equation

$$\frac{d^2 x^C}{dS^2} + \Gamma_{AB}^C u^A u^B = 0, \quad (2)$$

the parallel transport equation

$$\frac{dS^C}{dS} + \Gamma_{AB}^C S^A u^B = 0, \quad (3)$$

and the orthogonality condition

$$u^C S_C = 0. \quad (4)$$

These equations can be solved analytically without placing any restrictions on the components of S^C if the orbit is taken to be circular ($\theta = \pi/2$, $\dot{\theta} = \dot{r} = 0$) [15]. In the weak-field limit (i.e., dropping terms of second and higher order in $GM/c^2 r$) the spatial part of S^C is found to precess with an angular speed

$$\Omega = \sqrt{\frac{aGM}{cr_0^3}} \left[1 + \frac{3GM}{2c^2 r_0} (1 - a - b) \right], \quad (5)$$

¹ The properties of the induced matter are obtained by decomposing the 5D field equations $R_{AB} = 0$ into $\alpha\beta$ -, $\alpha\ell$ - and $\ell\ell$ -components. Requiring that the 4D field equations take their usual form, $G_{\alpha\beta} = (8\pi G/c^4)T_{\alpha\beta}$, one obtains an expression for the energy-momentum tensor $T_{\alpha\beta}$ of an induced 4D matter fluid that is a manifestation of pure geometry in 5D.

where r_0 is the distance between the test body (gyroscope) and the central mass (Earth).² The geodetic effect is the excess of Ω over the test body's *orbital* angular speed $\omega \equiv d\phi/dS$, which is found in the same limit to be

$$\omega = \sqrt{\frac{aGM}{cr_0^3}} \left[1 + \frac{GM}{2c^2r_0} (3 - b) \right]. \quad (6)$$

The accumulated geodetic precession angle per orbit, $\delta\phi = 2\pi(\omega - \Omega)/\omega$, is thus given to leading order by [15]

$$\delta\phi = \frac{3\pi GM}{c^2r_0} (1 + \Delta), \quad (7)$$

where the term in front of the parentheses on the left-hand side is the standard expression for geodetic precession in 4D GR, and

$$\Delta = a + \frac{2}{3}b \approx \frac{b}{6}, \quad (8)$$

is the predicted departure in 5D theory (in the last step, we have applied the above-noted consistency relation between a and b). Because the free parameter b is restricted to negative values, Kaluza-Klein gravity with the soliton metric can only accommodate precession rates *less than* those predicted by 4D GR. This is a common feature of most attempts to extend Einstein's theory with additional degrees of freedom, whether in the guise of extra dimensions or new scalar fields. Physically, this is consistent with the expectation that allowing the spin vector to wander into new regions of dynamical phase space can only reduce (not increase) the amount of precession that is observable in 4D. A conclusive experimental determination that $\Delta > 0$ might therefore be the basis for ruling out the theory. The GPB final results and implications for Δ and b are presented and discussed in Table 1 (Section 5).

4 Canonical metric

The line element for the canonical metric reads [9,10]:

$$dS^2 = \frac{\ell^2}{L^2} \left[B c^2 dt^2 - B^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] - d\ell^2, \quad (9)$$

where $B(r) \equiv 1 - 2GM/c^2r - r^2/L^2$ and L is a constant length scale, perhaps related to the cosmological constant via $\Lambda = 3/L^2$. As before, the equations of motions (2-4) can be solved analytically without placing any restrictions on S^A if we assume a circular orbit ($\theta = \pi/2, u^r = u^\theta = 0$) and model the GPB situation by placing the spin vector in the orbital plane, $S^\theta = 0$ with

² For simplicity we have set to zero a constant of the motion (k in [15]) associated with momentum along the extra dimension. This has the effect of "switching off" the S^ℓ -component for the soliton metric and might be worth revisiting in future work.

$rS^\phi \ll S^r$ (due to the choice of coordinates S^t, S^r and S^ℓ are dimensionless while S^θ and S^ϕ have dimensions of inverse length).

Carrying out this procedure in the same way as for the soliton metric, the spin angular velocity is found as

$$\Omega = \frac{c}{r_0} \sqrt{\frac{GM}{c^2 r_0} - \frac{r_0^2}{L^2}}. \quad (10)$$

Comparing to the orbital angular velocity, and using the metric to relate 5D and 4D proper distance, one finds [10] that the geodetic precession angle per orbit is again approximated in the weak-field limit ($r^2/L^2 \ll GM/c^2 r \ll 1$) by Eq. (7), but now with

$$\Delta = -\frac{2c^2 r_0^2 \mathcal{H}}{3GML}, \quad (11)$$

where $\mathcal{H} \equiv H_5/H_1 \cosh[(s_0 - s_m)/L]$ is a dimensionless combination of the normalized amplitudes of the spin vector [$S_A S^A = -(H_t^2 + H_r^2 + H_\ell^2)$] as well as the length scale L and two fiducial values of the 4D proper distance.

We do not have definitive values for all these parameters, but there are two clear routes to testing the theory. First, we can adopt the natural assumption that L is in fact related to the cosmological constant by $\Lambda = 3/L^2$ [9]. In observational cosmology it is usual to express Λ in terms of the normalized dark-energy density $\Omega_\Lambda \equiv \rho_\Lambda/\rho_{\text{crit}}$ where $\rho_\Lambda = \Lambda c^2/(8\pi G)$ and $\rho_{\text{crit}} = 3H_0^2/(8\pi G)$. When this is done, we can use experimental constraints on Δ to put an upper limit on the value of \mathcal{H} :

$$\mathcal{H} = -\frac{3GM}{2cH_0\sqrt{\Omega_\Lambda}r_0^2} \Delta. \quad (12)$$

Since \mathcal{H} is a dimensionless combination of a cosh term and a ratio of spin components, we expect it to have a positive value not too far from unity.

Alternatively, we can *assume* that \mathcal{H} is of order unity on naturalness grounds, and use the same relationship plus experimental constraints on Δ to put a *lower* limit on the value of the length scale L :

$$L = -\frac{2c^2 r_0^2}{3GM} \frac{1}{\Delta}. \quad (13)$$

Again, we would expect that L corresponds to a large distance, since the metric (9) deviates from 4D Schwarzschild by terms of order ℓ^2/L^2 . Its sign must also be positive (since it is a distance).

5 Summary and discussion

Constraints on Δ from GPB and implications for b, \mathcal{H} and L are presented in Table 1, where $\Delta = (r_{\text{NS}} - r_{\text{GR}})/r_{\text{GR}}$, r_{NS} is the measured north-south relativistic drift rate (with 1σ reported uncertainties) and $r_{\text{GR}} = -6606.1$ mas/yr is the general relativity prediction for geodetic precession given the actual

Table 1 GPB constraints on 5D metric parameters

Gyro	r_{NS} (mas/yr) [18]	$\Delta(\times 10^{-3})$	$ b _{\text{max}}$	$\mathcal{H}_{\text{max}}(\times 10^8)$	L_{min} (pc)
1	-6588.6 ± 31.7	$(-7.4, +2.2)$	0.045	1.5	32
2	-6707.0 ± 64.1	$(+5.6, +25.0)$	N/A	N/A	N/A
3	-6610.5 ± 43.2	$(-5.9, +7.2)$	0.043	1.2	41
4	-6588.7 ± 33.2	$(-7.7, +2.4)$	0.046	1.5	31
Joint	-6601.8 ± 18.3	$(-3.4, +2.1)$	0.020	0.7	71

GPB orbit [18]. We use $r_0 = 7018$ km [24] together with recent measurements of the Hubble parameter $H_0 = 74 \pm 2$ km s $^{-1}$ Mpc $^{-1}$ [25] and normalized dark-energy density $\Omega_\Lambda = 0.73 \pm 0.04$ from high-redshift supernovae [26] and the cosmic microwave background [27]. Our limits in each case come from the largest allowed *negative* values of Δ in Column 3 of Table 1.

These limits are consistent with pre-GPB expectations for both the soliton [15] and canonical metrics [28]. For example, using the joint confidence region for all four gyros and assuming that $\Lambda = 3/L^2$ for the canonical metric, we find that $\mathcal{H} \lesssim 7 \times 10^7$, implying that the component of test-body angular momentum along the ℓ -direction could be significantly larger than that along r (modulo a cosh-term). If instead we assume that $\mathcal{H} \sim 1$ then $L \gtrsim 70$ pc. These limits are complementary to somewhat stronger ones that can be obtained (with some additional assumptions) from cosmological tests, especially the magnitude-redshift relation for distant supernovae [29]. However, what is remarkable here is that there should be any sensitivity at all to cosmological parameters such as L or Λ in an experiment involving the motions of macroscopic test bodies in low-earth orbit (this appears to be unique to the canonical solution with its $(\ell/L)^2$ prefactor on the 4D part of the metric.) Similarly, our constraints for the soliton metric are complementary to stronger ones that can be placed on b for various solar-system bodies (including the Earth) using observational limits on violations of the equivalence principle [16]. The complementarity is nicely illustrated by the case of gyro 2, whose experimental range of uncertainty encompasses only positive values for Δ (i.e., geodetic precession rates greater than that predicted by GR). If this result had been confirmed by all four gyros, then the 5D theory would have been excluded using *either* choice of metric. In the event, it is thought that gyro 2 suffered from larger systematic effects than the others [30], and Kaluza-Klein gravity remains a viable extension of 4D general relativity.

The question is sometimes raised whether tests like this can ever conclusively establish the existence or non-existence of extra dimensions. Similar questions could perhaps be raised about other extrapolations from the current standard model. The best approach is to apply as many independent tests as possible and look for an accumulation of evidence that points to the same region of parameter space in the theory. Kaluza-Klein solitons have unique density profiles and other properties that make them viable dark-matter candidates [17], with theoretical arguments suggesting values of $|b| \sim 10^{-8} - 10^{-2}$ in the solar system and $|b|$ as large as ~ 0.1 in galaxy clusters [15]. If two or

more independent tests (such as gravitational lensing) were to converge on a nonzero value of b for the same system, it would not *prove* the existence of extra dimensions—one might look for scalar-field or other theories that could accommodate the same phenomena—but it would be a strong *prima facie* case. Moreover the example of the canonical metric shows that it is prudent to keep an open mind even in the case of a null result. With that metric no departure from 4D GR is expected for the classical tests; the effects of the extra dimension manifest themselves only for spinning test bodies. More than anything else, these remarks highlight the need for new solutions of the field equations in $D > 4$, and for a generalization of Birkhoff's theorem that could be used to discriminate between them in a compelling way.

Future work can build on these results in several ways. It would be of interest to test the predictions of Kaluza-Klein gravity for frame-dragging as well as geodetic precession. The case $k \neq 0$ for the soliton metric deserves further study. It may be possible to obtain stronger constraints for the canonical metric using violations of the equivalence principle [31]. Following the lead of Matsuno and Ishihara [32], it might also be useful to investigate precession effects for other 5D soliton-like solutions incorporating time-dependence [33], additional ℓ -dependence [34] and electric charge [35,36]. Finally, methods similar to those employed here can also be used to test other generalizations of 4D GR, like those that violate Lorentz invariance [37].

Acknowledgements J.M.O. thanks C.W.F. Everitt, R.J. Adler, A. Silbergleit and the other members of the Gravity Probe B theory group for discussions. R.D.E. acknowledges the Fisher College of Science and Mathematics at Towson University for travel support to present these results.

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