WRITTEN 8, MATH 369.101

Due: 12/12/2016

(1) Problem 3.8.13.

(2) Problems 3.8.20 and 3.8.21 from text. In these problems, $\mathbb{Z}\times\mathbb{Z}$ means the direct sum of $\mathbb{Z}$ (as a group under addition) with itself. The operation on the group is component-wise, so $(a,b)+(c,d) = (a+c, b+d)$. [Hint: For #20, look to use the Fundamental Theorem on Homomorphisms. For #21, consider adding an element in $\langle(2,2)\rangle$ to (a,b). What happens to the parity (being even,odd) of each component? Does this put a restriction on the kinds of elements in the coset for (a,b)?]

(3) Problem 3.8.25. [Hint: If N is any normal subgroup of a finite group G and the index $|G|/|N|$ is a prime number p, then $G/N \cong \mathbb{Z}_p$, regardless of any property that the group N may have had.]

(4) Consider the complex X shown below. Use the labels given for the vertices. For edges write $e_{i,j}$ for an edge between v_i and v_j, and for triangular faces write $f_{i,j,k}$ for a face that has vertices v_i, v_j and v_k.
   (a) Find three distinct elements of $C_0(X)$, all of which become equal (in the same coset) in the factor group $H_0(X)$.
   (b) Find three elements of $C_0(X)$ so that no non-zero combination of them is in the image of $\partial_1 : C_1(X) \to C_0(X)$.
   (c) Show that $H_0(X) \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$.
   (d) Find three independent elements of $C_1(X)$ which are in the kernel of $\partial_1 : C_1(X) \to C_0(X)$.
   (e) Show that $H_1(X) \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2$ and find a basis for it.

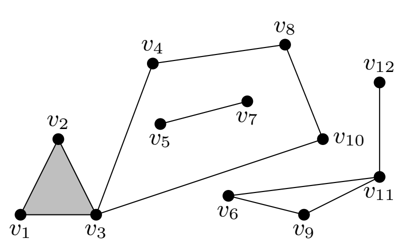

Figure 1. The complex X