WRITTEN 5, MATH 369.101

Due: 10/26/2016

(1) Suppose R is a commutative ring and that $r_1, r_2 \in R$. Show that the set
$$\{r_1 s_1 + r_2 s_2 \mid s_1, s_2 \in R\}$$
is an ideal of R.

(2) Recall that $\langle r \rangle$ is the set of all multiples of r in R (which is an ideal). Let $R = \mathbb{Z}_4 \oplus \mathbb{Z}_4$.
   (a) Write all the elements in the ideal $\langle (1,2) \rangle$.
   (b) Show that every element in $\mathbb{Z}_4 \oplus \mathbb{Z}_4$ is either in the coset $\langle (1,2) \rangle$ or in $(0,1) + \langle (1,2) \rangle$.

(3) Consider the ring T of 2×2 matrices $\begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$ with entries $a, b \in \mathbb{Z}$. Note that T is commutative.
   (a) Show that $\psi : \mathbb{Z}[x] \to T$ defined by
$$\psi\left(a_0 + a_1 x + \ldots + a_n x^n\right) = \begin{pmatrix} a_0 & a_1 \\ 0 & a_0 \end{pmatrix}$$
is a ring homomorphism.
   (b) Determine the kernel of ψ.
   (c) What factor ring is isomorphic to T?