WRITTEN 3, MATH 369.101

(1) Let $\mathbb{F} = \mathbb{Z}_5$ and let $p(x) = x^3+x+1$. Verify that $p(x)$ is an irreducible polynomial in $\mathbb{F}[x]$ and find the multiplicative inverse of $[x+1]$ in $\mathbb{F}[x]/\langle p(x)\rangle$.

(2) Prove that $\mathbb{Q}[x]/\langle x^2+1\rangle$ is a field. (Does a recent result help?)

(3) For some field $\mathbb{F}$ and $c \in \mathbb{F}$, consider the set of congruence classes $\mathbb{F}[x]/\langle x-c\rangle$. Prove that, given two polynomials $f(x), g(x)$ the equality $[f(x)] = [g(x)]$ occurs if and only if $f(c)$ and $g(c)$ are equal.

(4) (a) Write the congruence classes in $\mathbb{Z}_2[x]/\langle x^2+x+1\rangle$. Compute the addition and multiplication tables for these classes.

(b) Is there a bijective function from $\mathbb{Z}_2[x]/\langle x^2+x+1\rangle$ to the set of matrices in Example 4.1.3 that preserves the addition and multiplication? (If so, write what the bijection is; if not, explain why one doesn't exist.)

Date: Due: 09/26/2016.