WRITTEN 2, MATH 369.101

(1) Let $f(x) = 2x^4 - x^2$ and $g(x) = 2x^2 - x + 1$ in $\mathbb{Q}[x]$.
   (a) Find the quotient and remainder when dividing $f(x)$ by $g(x)$.
   (b) Find the greatest common divisor of $f(x)$ and $g(x)$.

(2) Choose some proof you know that there exists an infinite number of prime numbers. Using that proof as a model, prove that there are infinitely many irreducible polynomials over a field $\mathbb{F}$.

(3) The authors state that Theorem 4.2.9 is analogous to Theorem 1.2.7. Read these two theorems in the text and the proof of Theorem 1.2.7. Then write out a proof of Theorem 4.2.9, modeling it after the proof of Theorem 1.2.7 given in the book.

(4) How many irreducible polynomials of degree 2 are there in $\mathbb{Z}_7[x]$? Please state the number of irreducible polynomials and give a proof that you have counted all degree 2 polynomials in $\mathbb{Z}_7[x]$ that are irreducible.

   [Hint: If a degree 2 polynomial is not irreducible, then how can you write it?]

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