WRITTEN 1, MATH 369.101

(1) Prove that the set $\mathbb{Q}(\sqrt{7}) := \{a + b\sqrt{7} \mid a, b \in \mathbb{Q}\}$ is a field under the operations inherited from $\mathbb{R}$.
   [Take some care with the closure axiom and the existence of inverses. For the other axioms, only briefly explain why they hold.]

(2) Prove that $\mathbb{Q}(\sqrt{7})$ is the smallest subfield of $\mathbb{R}$ that contains $\sqrt{7}$.

(3) Describe the smallest subfield of $\mathbb{R}$ containing $\sqrt[3]{7}$.
   [Think of this problem in three parts: first figure out a set description of what you think the subfield is; then prove that it is a field under addition and multiplication inherited from $\mathbb{R}$, similar to #1; then prove it is the smallest subfield containing $\sqrt[3]{7}$, as in #2.]

(4) Produce addition and multiplication tables for a field having 4 elements.