PREP 1

Reading. (BB) The first part of section 4.1, up to Proposition 4.1.5. As with all reading assignments, be prepared to answer questions about the reading.

Exercises. Work out the following problems. For numbers 1–3, if the object is not a field, explain which axiom of fields does not fit the example. If it is a field be ready to prove that the axioms hold, in particular the axiom on inverses.

(1) Is $\mathbb{Z}_{11}$, with addition and multiplication modulo 11, a field?
(2) Using matrix addition and multiplication, is the set of 2×2 matrices with real entries and non-zero determinant a field?
(3) Is $\mathbb{Z}_9$, with addition and multiplication modulo 9, a field?
(4) Prove that $\mathbb{Q}$, the set of rational numbers, is the smallest subfield in the field of real numbers.