EXTRA PROBLEMS, MATH 369.101

Due: 10/10/2016

You may complete the following problems for extra points towards your homework grade. The number of points you get added to your grade is listed before the problem number.

(4 points) 1. Explain why the following statement is true: "If $f(x)$ is a monic polynomial with integer coefficients, and r is a rational number with $f(r) = 0$, then r must be an integer." Then find all integer roots of $f(x) = x^3 - 10x^2 + 27x - 18$.

(4 points) 2. For which values of $a = 1, 2, 3, 4$ is $\mathbb{Z}_5[x]/\langle x^2 + a\rangle$ a field? Show your work.

(8 points) 3. Show that the set of all 2×2 matrices with entries in $\mathbb{Q}$ that have the form $\begin{bmatrix} a & b \\ 17b & a \end{bmatrix}$ is a field. With which other numbers could we replace 17 in the statement and still have a field?

(4 points) 4. Prove that $\mathbb{R}[x]/\langle x^2 + 2\rangle$ is isomorphic to $\mathbb{C}$.

(4 points) 5. Find an infinite set of integers n such that $p(x) = x^2 + 100x + n$ is irreducible over $\mathbb{Q}$ (include a proof that $p(x)$ is irreducible).

(8 points) 6. The goal of this problem is to find the multiplicative inverse of $[x^2 + 2x + 2]$ in $\mathbb{Q}[x]/\langle x^4 + 1\rangle$. Follow the steps below to do so.

(a) Show that $\gcd(x^4 + 1, x^2 + 2x + 2) = 1$.

(b) While finding the gcd, you should have done division of $f(x) = x^4 + 1$ by $g(x) = x^2 + 2x + 2$. Use this to write the remainder as a combination of $f(x)$ and $g(x)$ (think about what the Division Algorithm says).

(c) Use your answer above to figure out the inverse of $[x^2 + 2x + 2]$ in $\mathbb{Q}[x]/\langle x^4 + 1\rangle$.